Measuring Masking Fault-Tolerance

Pablo F. Castro, Pedro R. D'Argenio, Ramiro Demasi, Luciano Putruele

Dependable Systems dTime Talks
October, 2020

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Investigaciones Científicas y Técnicas (CONICET),

Motivation

```
module NOMINAL

  b : [0..1] init 0;

  [w0]   true -> (b' = 0);
  [w1]   true -> (b' = 1);
  [r0]   b=0  -> true;
  [r1]   b=1  -> true;

endmodule
```

Ideal behaviour

```
module FAULTY

  v : [0..3] init 0;

  [w0]     true -> (v' = 0);
  [w1]     true -> (v' = 3);
  [r0]     v<=1 -> true;
  [r1]     v>=2 -> true;
  [fault] v<3  -> (v' = v+1);
  [fault] v>0  -> (v' = v-1);

endmodule
```

Redundancy

Behaviour of the implementation

A fault is masked when the occurrence of it have no observable consequences

Motivation

```
module NOMINAL

  b : [0..1] init 0;

  [w0]   true -> (b' = 0);
  [w1]   true -> (b' = 1);
  [r0]   b=0  -> true;
  [r1]   b=1  -> true;

endmodule
```

```
module FAULTY

  v : [0..3] init 0;

  [w0]     true -> (v' = 0);
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  [r0]     v<=1 -> true;
  [r1]     v>=2 -> true;
  [fault] v<3  -> (v' = v+1);
  [fault] v>0  -> (v' = v-1);

endmodule
```

- ¿Can an implementation mask all faults?

Motivation

```
module NOMINAL

  b : [0..1] init 0;

  [w0]   true -> (b' = 0);
  [w1]   true -> (b' = 1);
  [r0]   b=0  -> true;
  [r1]   b=1  -> true;

endmodule
```

```
module FAULTY

  v : [0..5] init 0;

  [w0]     true -> (v' = 0);
  [w1]     true -> (v' = 5);
  [r0]     v<=2 -> true;
  [r1]     v>=3 -> true;
  [fault] v<5  -> (v' = v+1);
  [fault] v>0  -> (v' = v-1);

endmodule
```

- ¿Can an implementation mask all faults?

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Motivation

```
module NOMINAL

  b : [0..1] init 0;

  [w0]   true -> (b' = 0);
  [w1]   true -> (b' = 1);
  [r0]   b=0  -> true;
  [r1]   b=1  -> true;

endmodule
```

```
module FAULTY

  v : [0..5] init 0;

  [w0]     true -> (v' = 0);
  [w1]     true -> (v' = 5);
  [r0]     v<=2 -> true;
  [r1]     v>=3 -> true;
  [fault] v<5  -> (v' = v+1);
  [fault] v>0  -> (v' = v-1);

endmodule
```

- ¿Can an implementation mask all faults?
- Given two implementations ¿can we determine which is better on masking?

Motivation

```
module NOMINAL

  b : [0..1] init 0;

  [w0]   true -> (b' = 0);
  [w1]   true -> (b' = 1);
  [r0]   b=0  -> true;
  [r1]   b=1  -> true;

endmodule
```

```
module FAULTY

  v : [0..5] init 0;

  [w0]     true -> (v' = 0);
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  [r0]     v<=2 -> true;
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  [fault] v<5  -> (v' = v+1);
  [fault] v>0  -> (v' = v-1);

endmodule
```

- ¿Can an implementation mask all faults?
  - Behavioural relation
  - Game characterisation
  - Algorithm
- Given two implementations ¿can we determine which is better on masking?
  - Game based distance
  - Algorithm
  - Tool

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Strong Masking Simulation

Definition 3.1. Let $A = \langle S, \Sigma, \rightarrow, s_0 \rangle$ and $A' = \langle S', \Sigma_{\mathcal{F}}, \rightarrow', s'_0 \rangle$ be two transition systems. A' is *strong masking fault-tolerant* with respect to A if there exists a relation $\mathbf{M} \subseteq S \times S'$ between A and A' such that:

(A) $s_0 \, \mathbf{M} \, s'_0$, and

(B) for all $s \in S, s' \in S'$ with $s \, \mathbf{M} \, s'$ and all $e \in \Sigma$ the following holds:

  (1) if $s \xrightarrow{e} t$ then $\exists \, t' \in S' : s' \xrightarrow{e}' t' \wedge t \, \mathbf{M} \, t'$;

  (2) if $s' \xrightarrow{e}' t'$ then $\exists \, t \in S : s \xrightarrow{e} t \wedge t \, \mathbf{M} \, t'$;

  (3) if $s' \xrightarrow{F}' t'$ for some $F \in \mathcal{F}$ then $s \, \mathbf{M} \, t'$.

If such a relation exists we say that A' is a *strong masking fault-tolerant implementation* of A, denoted by $A \preceq_m A'$.

Strong Masking Simulation

Nominal: no faults

Implementation: has faults

Definition 3.1. Let $A = \langle S, \Sigma, \rightarrow, s_0 \rangle$ and $A' = \langle S', \Sigma_{\mathcal{F}}, \rightarrow', s'_0 \rangle$ be two transition systems. A' is *strong masking fault-tolerant* with respect to A if there exists a relation $\mathbf{M} \subseteq S \times S'$ between A and A' such that:

(A) $s_0 \,\mathbf{M}\, s'_0$, and

(B) for all $s \in S, s' \in S'$ with $s \,\mathbf{M}\, s'$ and all $e \in \Sigma$ the following holds:

  (1) if $s \xrightarrow{e} t$ then $\exists\, t' \in S' : s' \xrightarrow{e}{}' t' \wedge t \,\mathbf{M}\, t'$;

  (2) if $s' \xrightarrow{e}{}' t'$ then $\exists\, t \in S : s \xrightarrow{e} t \wedge t \,\mathbf{M}\, t'$;

  (3) if $s' \xrightarrow{F}{}' t'$ for some $F \in \mathcal{F}$ then $s \,\mathbf{M}\, t'$.

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Strong Masking Simulation

Implementation:
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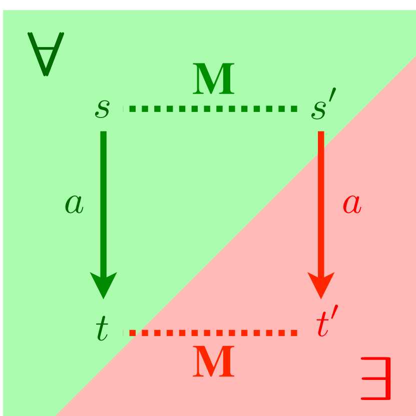
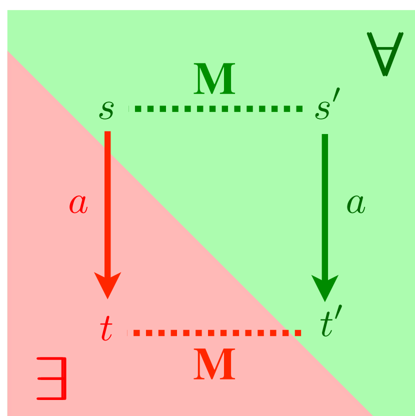
Just like
bisimulation

If such a relation exists we say that A' is a *strong masking fault-tolerant implementation* of A, denoted by $A \preceq_m A'$.





Nominal: no faults

Strong Masking Simulation

Implementation: has faults

Definition 3.1. Let $A = \langle S, \Sigma, \rightarrow, s_0 \rangle$ and $A' = \langle S', \Sigma_{\mathcal{F}}, \rightarrow', s'_0 \rangle$ be two transition systems. A' is *strong masking fault-tolerant* with respect to A if there exists a relation $\mathbf{M} \subseteq S \times S'$ between A and A' such that:

(A) $s_0 \,\mathbf{M}\, s'_0$, and

(B) for all $s \in S, s' \in S'$ with $s \,\mathbf{M}\, s'$ and all $e \in \Sigma$ the following holds:

(1) if $s \xrightarrow{e} t$ then $\exists\, t' \in S' : s' \xrightarrow{e}{}' t' \wedge t \,\mathbf{M}\, t'$;

(2) if $s' \xrightarrow{e}{}' t'$ then $\exists\, t \in S : s \xrightarrow{e} t \wedge t \,\mathbf{M}\, t'$;

(3) if $s' \xrightarrow{F}{}' t'$ for some $F \in \mathcal{F}$ then $s \,\mathbf{M}\, t'$.

Just like bisimulation

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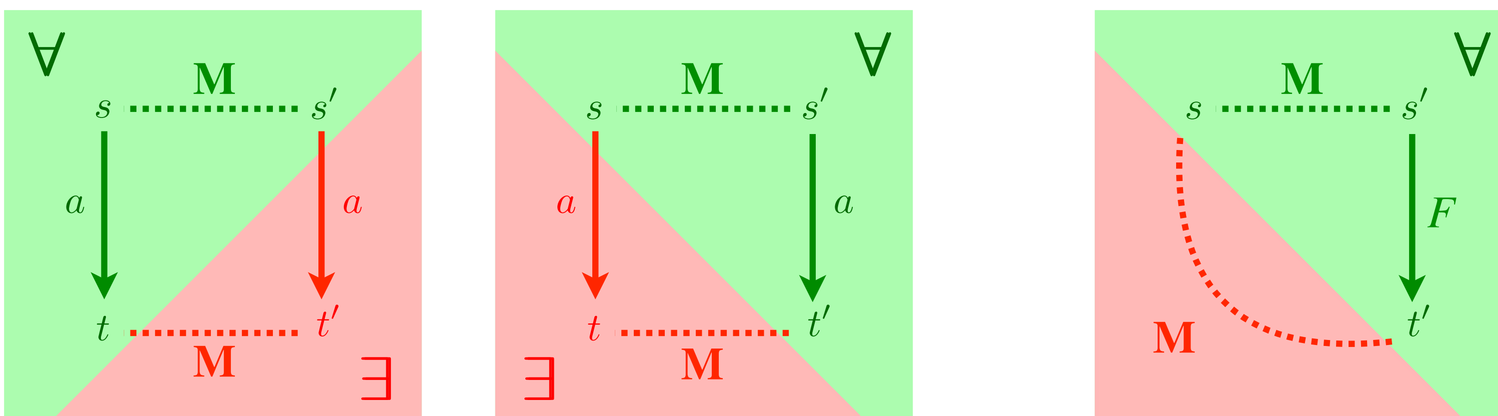

Strong Masking Simulation

Nominal: no faults

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(A) $s_0 \,\mathbf{M}\, s'_0$, and

(B) for all $s \in S, s' \in S'$ with $s \,\mathbf{M}\, s'$ and all $e \in \Sigma$ the following holds:

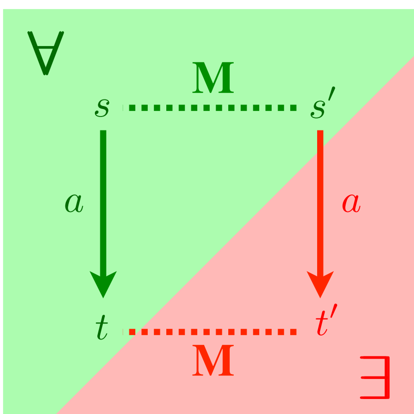
(1) if $s \xrightarrow{e} t$ then $\exists\, t' \in S' : s' \xrightarrow{e}{}' t' \wedge t \,\mathbf{M}\, t'$;

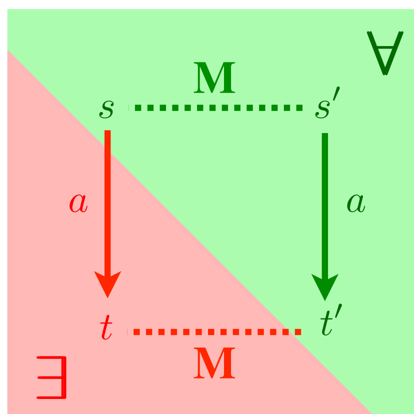
(2) if $s' \xrightarrow{e}{}' t'$ then $\exists\, t \in S : s \xrightarrow{e} t \wedge t \,\mathbf{M}\, t'$;

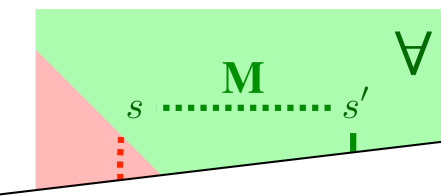
(3) if $s' \xrightarrow{F}{}' t'$ for some $F \in \mathcal{F}$ then $s \,\mathbf{M}\, t'$.

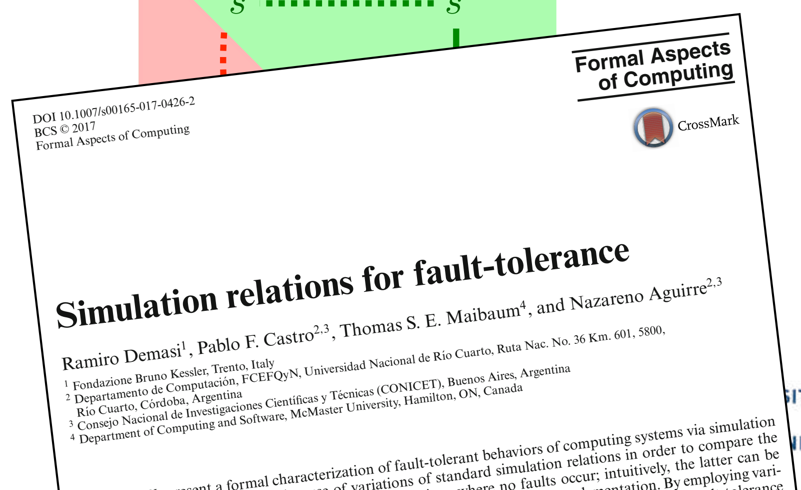
Just like bisimulation

If such a relation exists we say that A' is a *strong masking fault-tolerant implementation* of A, denoted by $A \preceq_m A'$.





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Simulation relations for fault-tolerance

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[4] Department of Computing and Software, McMaster University, Hamilton, ON, Canada

Abstract. We present a formal characterization of fault-tolerant behaviors of computing systems via simulation

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Strong Masking Simulation

```
module NOMINAL

  b : [0..1] init 0;

  [w0]   true -> (b' = 0);
  [w1]   true -> (b' = 1);
  [r0]   b=0  -> true;
  [r1]   b=1  -> true;

endmodule
```

```
module FAULTY

  v : [0..3] init 0;

  [w0]     true -> (v' = 0);
  [w1]     true -> (v' = 3);
  [r0]     v<=1 -> true;
  [r1]     v>=2 -> true;
  [fault] v<3  -> (v' = v+1);
  [fault] v>0  -> (v' = v-1);

endmodule
```

$$\texttt{NOMINAL} \not\preceq_m \texttt{FAULTY}$$

Strong Masking Simulation

```
module NOMINAL

  b : [0..1] init 0;

  [w0]   true -> (b' = 0);
  [w1]   true -> (b' = 1);
  [r0]   b=0  -> true;
  [r1]   b=1  -> true;

endmodule
```

```
module FAULTY_BOUNDED

  v : [0..3] init 0;
  f : [0..1] init 0;

  [w0]    true -> (v' = 0);
  [w1]    true -> (v' = 3);
  [r0]    v<=1 -> true;
  [r1]    v>=2 -> true;
  [fault] (v<3) & (f<1) -> (v' = v+1) &
                           (f' = f+1);
  [fault] (v>0) & (f<1) -> (v' = v-1) &
                           (f' = f+1);

endmodule
```

$$\mathbf{M} = \{\langle b, (v, f)\rangle \mid 2b \leq v \leq 2b + 1\}$$

$$\texttt{NOMINAL} \preceq_m \texttt{FAULTY\_BOUNDED}$$

Weak Masking Simulation

Definition 3.2. Let $A = \langle S, \Sigma, \rightarrow, s_0 \rangle$ and $A' = \langle S', \Sigma_{\mathcal{F}}, \rightarrow', s'_0 \rangle$ be two transition systems with Σ possibly containing τ. A' is *weak masking fault-tolerant* with respect to A if there is a relation $\mathbf{M} \subseteq S \times S'$ between A and A' such that:

(A) $s_0 \, \mathbf{M} \, s'_0$

(B) for all $s \in S, s' \in S'$ with $s \, \mathbf{M} \, s'$ and all $e \in \Sigma \cup \{\tau\}$ the following holds:

(1) if $s \xrightarrow{e} t$ then $\exists \, t' \in S' : s' \stackrel{e}{\Rightarrow}' t' \wedge t \, \mathbf{M} \, t'$;

(2) if $s' \xrightarrow{e}' t'$ then $\exists \, t \in S : s \stackrel{e}{\Rightarrow} t \wedge t \, \mathbf{M} \, t'$;

(3) if $s' \xrightarrow{F}' t'$ for some $F \in \mathcal{F}$ then $s \, \mathbf{M} \, t'$.

If such a relation exists, we say that A' is a *weak masking fault-tolerant implementation* of A, denoted by $A \preceq^w_m A'$.

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Weak Masking Simulation

Equivalently

Definition 3.2. Let $A = \langle S, \Sigma, \rightarrow, s_0 \rangle$ and $A' = \langle S', \Sigma_{\mathcal{F}}, \rightarrow', s'_0 \rangle$ be two transition systems with Σ possibly containing τ. A' is *weak masking fault-tolerant* with respect to A if there is a relation $\mathbf{M} \subseteq S \times S'$ between A and A' such that:

(A) $s_0 \, \mathbf{M} \, s'_0$

(B) for all $s \in S, s' \in S'$ with $s \, \mathbf{M} \, s'$ and all $e \in \Sigma \cup \{\tau\}$ the following holds:

  (1) if $s \stackrel{e}{\Rightarrow} t$ then $\exists\, t' \in S' : s' \stackrel{e}{\Rightarrow}' t' \wedge t \, \mathbf{M} \, t'$;

  (2) if $s' \stackrel{e}{\Rightarrow}' t'$ then $\exists\, t \in S : s \stackrel{e}{\Rightarrow} t \wedge t \, \mathbf{M} \, t'$;

  (3) if $s' \xrightarrow{F}' t'$ for some $F \in \mathcal{F}$ then $s \, \mathbf{M} \, t'$.

If such a relation exists, we say that A' is a *weak masking fault-tolerant implementation* of A, denoted by $A \preceq_m^w A'$.

Hence,
every result for strong also applies to weak by replacing de strong transition relation by the weak one (except for faults)

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Masking Simulation Game

Definition 3.5. Let $A = \langle S, \Sigma, \rightarrow, s_0 \rangle$ and $A' = \langle S', \Sigma_{\mathcal{F}}, \rightarrow', s'_0 \rangle$ two transition systems. The *strong masking game graph* $\mathcal{G}_{A,A'} = \langle V^G, V_{\mathrm{R}}, V_{\mathrm{V}}, E^G, {v_0}^G \rangle$ for two players is defined as follows:

- $V^G = (S \times (\Sigma^1 \cup \Sigma^2_{\mathcal{F}} \cup \{\#\}) \times S' \times \{\mathrm{R}, \mathrm{V}\}) \cup \{v_{err}\}$
- The initial state is $v_0^G = \langle s_0, \#, s'_0, \mathrm{R} \rangle$, where the Refuter starts playing
- The Refuter's states are $V_{\mathrm{R}} = \{(s, \#, s', \mathrm{R}) \mid s \in S \wedge s' \in S'\} \cup \{v_{err}\}$
- The Verifier's states are $V_{\mathrm{V}} = \{(s, \sigma, s', \mathrm{V}) \mid s \in S \wedge s' \in S' \wedge \sigma \in (\Sigma^1 \cup \Sigma^2_{\mathcal{F}})\}$

and E^G is the minimal set satisfying:

- $\{((s, \#, s', \mathrm{R}), (t, \sigma^1, s', \mathrm{V})) \mid \exists\, \sigma \in \Sigma : s \xrightarrow{\sigma} t\} \subseteq E^G$,
- $\{((s, \#, s', \mathrm{R}), (s, \sigma^2, t', \mathrm{V})) \mid \exists\, \sigma \in \Sigma_{\mathcal{F}} : s' \xrightarrow{\sigma}' t'\} \subseteq E^G$,
- $\{((s, \sigma^2, s', \mathrm{V}), (t, \#, s', \mathrm{R})) \mid \exists\, \sigma \in \Sigma : s \xrightarrow{\sigma} t\} \subseteq E^G$,
- $\{((s, \sigma^1, s', \mathrm{V}), (s, \#, t', \mathrm{R})) \mid \exists\, \sigma \in \Sigma : s' \xrightarrow{\sigma}' t'\} \subseteq E^G$,
- $\{((s, F^2, s', \mathrm{V}), (s, \#, s', \mathrm{R}))\} \subseteq E^G$, for any $F \in \mathcal{F}$.
- If there is no outgoing transition from some state v, then, we additionally assume $(v, v_{err}) \in E^G$ and $(v_{err}, v_{err}) \in E^G$.

Masking Simulation Game

Definition 3.5. Let $A = \langle S, \Sigma, \rightarrow, s_0 \rangle$ and $A' = \langle S', \Sigma_{\mathcal{F}}, \rightarrow', s'_0 \rangle$ two transition systems. The *strong masking game graph* $\mathcal{G}_{A,A'} = \langle V^G, V_{\mathrm{R}}, V_{\mathrm{V}}, E^G, {v_0}^G \rangle$ for two players is defined as follows:

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- $\{((s, \#, s', \mathrm{R}), (s, \sigma^2, t', \mathrm{V})) \mid \exists\, \sigma \in \Sigma_{\mathcal{F}} : s' \xrightarrow{\sigma}{}' t'\} \subseteq E^G$,
- $\{((s, \sigma^2, s', \mathrm{V}), (t, \#, s', \mathrm{R})) \mid \exists\, \sigma \in \Sigma : s \xrightarrow{\sigma} t\} \subseteq E^G$,
- $\{((s, \sigma^1, s', \mathrm{V}), (s, \#, t', \mathrm{R})) \mid \exists\, \sigma \in \Sigma : s' \xrightarrow{\sigma}{}' t'\} \subseteq E^G$,
- $\{((s, F^2, s', \mathrm{V}), (s, \#, s', \mathrm{R}))\} \subseteq E^G$, for any $F \in \mathcal{F}$.
- If there is no outgoing transition from some state v, then, we additionally assume $(v, v_{err}) \in E^G$ and $(v_{err}, v_{err}) \in E^G$.

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Masking Simulation Game

Definition 3.5. Let $A = \langle S, \Sigma, \rightarrow, s_0 \rangle$ and $A' = \langle S', \Sigma_{\mathcal{F}}, \rightarrow', s'_0 \rangle$ two transition systems. The *strong masking game graph* $\mathcal{G}_{A,A'} = \langle V^G, V_{\mathrm{R}}, V_{\mathrm{V}}, E^G, {v_0}^G \rangle$ for two players is defined as follows:

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- $\{((s, \#, s', \mathrm{R}), (s, \sigma^2, t', \mathrm{V})) \mid \exists\, \sigma \in \Sigma_{\mathcal{F}} : s' \xrightarrow{\sigma}{}' t'\} \subseteq E^G$,
- $\{((s, \sigma^2, s', \mathrm{V}), (t, \#, s', \mathrm{R})) \mid \exists\, \sigma \in \Sigma : s \xrightarrow{\sigma} t\} \subseteq E^G$,
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- $\{((s, F^2, s', \mathrm{V}), (s, \#, s', \mathrm{R}))\} \subseteq E^G$, for any $F \in \mathcal{F}$.
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Masking Simulation Game

Definition 3.5. Let $A = \langle S, \Sigma, \rightarrow, s_0 \rangle$ and $A' = \langle S', \Sigma_{\mathcal{F}}, \rightarrow', s'_0 \rangle$ two transition systems. The *strong masking game graph* $\mathcal{G}_{A,A'} = \langle V^G, V_{\mathrm{R}}, V_{\mathrm{V}}, E^G, {v_0}^G \rangle$ for two players is defined as follows:

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- The Verifier's states are $V_{\mathrm{V}} = \{(s, \sigma, s', \mathrm{V}) \mid s \in S \wedge s' \in S' \wedge \sigma \in (\Sigma^1 \cup \Sigma^2_{\mathcal{F}})\}$

and E^G is the minimal set satisfying:

- $\{((s, \#, s', \mathrm{R}), (t, \sigma^1, s', \mathrm{V})) \mid \exists\, \sigma \in \Sigma : s \xrightarrow{\sigma} t\} \subseteq E^G$,
- $\{((s, \#, s', \mathrm{R}), (s, \sigma^2, t', \mathrm{V})) \mid \exists\, \sigma \in \Sigma_{\mathcal{F}} : s' \xrightarrow{\sigma}' t'\} \subseteq E^G$,
- $\{((s, \sigma^2, s', \mathrm{V}), (t, \#, s', \mathrm{R})) \mid \exists\, \sigma \in \Sigma : s \xrightarrow{\sigma} t\} \subseteq E^G$,
- $\{((s, \sigma^1, s', \mathrm{V}), (s, \#, t', \mathrm{R})) \mid \exists\, \sigma \in \Sigma : s' \xrightarrow{\sigma}' t'\} \subseteq E^G$,
- $\{((s, F^2, s', \mathrm{V}), (s, \#, s', \mathrm{R}))\} \subseteq E^G$, for any $F \in \mathcal{F}$.
- If there is no outgoing transition from some state v, then, we additionally assume $(v, v_{err}) \in E^G$ and $(v_{err}, v_{err}) \in E^G$.

Masking Simulation Game

Definition 3.5. Let $A = \langle S, \Sigma, \rightarrow, s_0 \rangle$ and $A' = \langle S', \Sigma_{\mathcal{F}}, \rightarrow', s'_0 \rangle$ two transition systems. The *strong masking game graph* $\mathcal{G}_{A,A'} = \langle V^G, V_{\mathrm{R}}, V_{\mathrm{V}}, E^G, {v_0}^G \rangle$ for two players is defined as follows:

- $V^G = (S \times (\Sigma^1 \cup \Sigma^2_{\mathcal{F}} \cup \{\#\}) \times S' \times \{\mathrm{R}, \mathrm{V}\}) \cup \{v_{err}\}$
- The initial state is $v_0^G = \langle s_0, \#, s'_0, \mathrm{R} \rangle$, where the Refuter starts playing
- The Refuter's states are $V_{\mathrm{R}} = \{(s, \#, s', \mathrm{R}) \mid s \in S \wedge s' \in S'\} \cup \{v_{err}\}$
- The Verifier's states are $V_{\mathrm{V}} = \{(s, \sigma, s', \mathrm{V}) \mid s \in S \wedge s' \in S' \wedge \sigma \in (\Sigma^1 \cup \Sigma^2_{\mathcal{F}})\}$

and E^G is the minimal set satisfying:

- $\{((s, \#, s', \mathrm{R}), (t, \sigma^1, s', \mathrm{V})) \mid \exists\, \sigma \in \Sigma : s \xrightarrow{\sigma} t\} \subseteq E^G$,
- $\{((s, \#, s', \mathrm{R}), (s, \sigma^2, t', \mathrm{V})) \mid \exists\, \sigma \in \Sigma_{\mathcal{F}} : s' \xrightarrow{\sigma}{}' t'\} \subseteq E^G$,
- $\{((s, \sigma^2, s', \mathrm{V}), (t, \#, s', \mathrm{R})) \mid \exists\, \sigma \in \Sigma : s \xrightarrow{\sigma} t\} \subseteq E^G$,
- $\{((s, \sigma^1, s', \mathrm{V}), (s, \#, t', \mathrm{R})) \mid \exists\, \sigma \in \Sigma : s' \xrightarrow{\sigma}{}' t'\} \subseteq E^G$,
- $\{((s, F^2, s', \mathrm{V}), (s, \#, s', \mathrm{R}))\} \subseteq E^G$, for any $F \in \mathcal{F}$.
- If there is no outgoing transition from some state v, then, we additionally assume $(v, v_{err}) \in E^G$ and $(v_{err}, v_{err}) \in E^G$.

Masking Simulation Game

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- The initial state is $v_0^G = \langle s_0, \#, s'_0, \mathrm{R} \rangle$, where the Refuter starts playing
- The Refuter's states are $V_{\mathrm{R}} = \{(s, \#, s', \mathrm{R}) \mid s \in S \wedge s' \in S'\} \cup \{v_{err}\}$
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and E^G is the minimal set satisfying:

- $\{((s, \#, s', \mathrm{R}), (t, \sigma^1, s', \mathrm{V})) \mid \exists\, \sigma \in \Sigma : s \xrightarrow{\sigma} t\} \subseteq E^G$,
- $\{((s, \#, s', \mathrm{R}), (s, \sigma^2, t', \mathrm{V})) \mid \exists\, \sigma \in \Sigma_{\mathcal{F}} : s' \xrightarrow{\sigma}' t'\} \subseteq E^G$,
- $\{((s, \sigma^2, s', \mathrm{V}), (t, \#, s', \mathrm{R})) \mid \exists\, \sigma \in \Sigma : s \xrightarrow{\sigma} t\} \subseteq E^G$,
- $\{((s, \sigma^1, s', \mathrm{V}), (s, \#, t', \mathrm{R})) \mid \exists\, \sigma \in \Sigma : s' \xrightarrow{\sigma}' t'\} \subseteq E^G$,
- $\{((s, F^2, s', \mathrm{V}), (s, \#, s', \mathrm{R}))\} \subseteq E^G$, for any $F \in \mathcal{F}$.
- If there is no outgoing transition from some state v, then, we additionally assume $(v, v_{err}) \in E^G$ and $(v_{err}, v_{err}) \in E^G$.

We are in the presence of a masking simulation iff the Verifier has a winning strategy (i.e. the Refuter is not able to lead the Verifier to the error state)

Masking Simulation Game (Algorithm)

Definition 3.9. Given a strong masking game graph $\mathcal{G}_{A,A'}$, the sets U_i^j (for $i, j \geq 0$) are defined as follows:

$$U_i^0 = U_0^j = \emptyset,$$

$$U_1^1 = \{v_{err}\},$$

$$U_{i+1}^{j+1} = \{v' \mid v' \in V_{\mathrm{R}} \wedge \mathrm{post}(v') \cap U_{i+1}^j \neq \emptyset\}$$
$$\cup \{v' \mid v' \in V_{\mathrm{V}} \wedge \mathrm{post}(v') \subseteq \bigcup_{i' \leq i+1, j' \leq j} U_{i'}^{j'} \wedge \mathrm{post}(v') \cap U_{i+1}^j \neq \emptyset \wedge \mathrm{pr}_1(v') \notin \mathcal{F}\}$$
$$\cup \{v' \mid v' \in V_{\mathrm{V}} \wedge \mathrm{post}(v') \subseteq \bigcup_{i' \leq i, j' \leq j} U_{i'}^{j'} \wedge \mathrm{post}(v') \cap U_i^j \neq \emptyset \wedge \mathrm{pr}_1(v') \in \mathcal{F}\}$$

Furthermore, $U^k = \bigcup_{i \geq 0} U_i^k$ and $U = \bigcup_{k \geq 0} U^k$.

Fix-point calculation

Lemma 3.10. *The Refuter has a winning strategy in $\mathcal{G}_{A,A'}$ (or $\mathcal{G}^W_{A,A'}$) iff $v_0^G \in U$*

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Back to the example

Which solution is better?

```
module NOMINAL

  b : [0..1] init 0;

  [w0]   true -> (b' = 0);
  [w1]   true -> (b' = 1);
  [r0]   b=0  -> true;
  [r1]   b=1  -> true;

endmodule
```

```
module FAULTY

  v : [0..3] init 0;

  [w0]     true -> (v' = 0);
  [w1]     true -> (v' = 3);
  [r0]     v<=1 -> true;
  [r1]     v>=2 -> true;
  [fault] v<3  -> (v' = v+1);
  [fault] v>0  -> (v' = v-1);

endmodule
```

```
module FAULTY

  v : [0..5] init 0;

  [w0]     true -> (v' = 0);
  [w1]     true -> (v' = 5);
  [r0]     v<=2 -> true;
  [r1]     v>=3 -> true;
  [fault] v<5  -> (v' = v+1);
  [fault] v>0  -> (v' = v-1);

endmodule
```

Back to the example

```
module NOMINAL

  b : [0..1] init 0;

  [w0]   true -> (b' = 0);
  [w1]   true -> (b' = 1);
  [r0]   b=0  -> true;
  [r1]   b=1  -> true;

endmodule
```

Which solution is better?

```
module FAULTY_BOUNDED

  v : [0..3] init 0;
  f : [0..1] init 0;

  [w0]    true -> (v' = 0);
  [w1]    true -> (v' = 3);
  [r0]    v<=1 -> true;
  [r1]    v>=2 -> true;
  [fault] (v<3) & (f<1) -> (v' = v+1) &
                           (f' = f+1);
  [fault] (v>0) & (f<1) -> (v' = v-1) &
                           (f' = f+1);

endmodule
```

```
module FAULTY_BOUNDED

  v : [0..5] init 0;
  f : [0..2] init 0;

  [w0]    true -> (v' = 0);
  [w1]    true -> (v' = 5);
  [r0]    v<=2 -> true;
  [r1]    v>=3 -> true;
  [fault] (v<3) & (f<2) -> (v' = v+1) &
                           (f' = f+1);
  [fault] (v>0) & (f<2) -> (v' = v-1) &
                           (f' = f+1);

endmodule
```

Add the counting artifact and check masking simulation

Back to the example

Which solution is better?

Needless to say that this is an ad-hoc solution and prone to error

```
module NOMINAL

  b : [0..1] init 0;

  [w0]   true -> (b' = 0);
  [w1]   true -> (b' = 1);
  [r0]   b=0  -> true;
  [r1]   b=1  -> true;

endmodule
```

```
module FAULTY_BOUNDED

  v : [0..3] init 0;
  f : [0..1] init 0;

  [w0]     true -> (v' = 0);
  [w1]     true -> (v' = 3);
  [r0]     v<=1 -> true;
  [r1]     v>=2 -> true;
  [fault] (v<3) & (f<1) -> (v' = v+1) &
                           (f' = f+1);
  [fault] (v>0) & (f<1) -> (v' = v-1) &
                           (f' = f+1);

endmodule
```

```
module FAULTY_BOUNDED

  v : [0..5] init 0;
  f : [0..2] init 0;

  [w0]     true -> (v' = 0);
  [w1]     true -> (v' = 5);
  [r0]     v<=2 -> true;
  [r1]     v>=3 -> true;
  [fault] (v<3) & (f<2) -> (v' = v+1) &
                           (f' = f+1);
  [fault] (v>0) & (f<2) -> (v' = v-1) &
                           (f' = f+1);

endmodule
```

Add the counting artifact and check masking simulation

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Quantitative Masking Game

The quantitaive masking game $\mathcal{Q}_{A,A'}$ is defined by extending the masking game with the reward function

$$\mathrm{r}((s,\sigma,s',X)) = \begin{cases} (1,0) & \text{if } \sigma \in \mathcal{F} \\ (0,0) & \text{otherwise} \end{cases} \qquad \mathrm{r}(v_{err}) = (0,1)$$

Take a play $\rho = \rho_0\rho_1\rho_2,\ldots$ and let $\mathrm{r}(\rho_i) = (a_i, b_i)$ for all $i \geq 0$. We define the masking payoff function by:

$$f_m(\rho) = \lim_{n\to\infty} \frac{b_n}{1 + \sum_{i=0}^{n} a_i}$$

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Take a play $\rho = \rho_0\rho_1\rho_2,\dots$ and let $\mathrm{r}(\rho_i) = (a_i, b_i)$ for all $i \geq 0$. We define the masking payoff function by:

$$f_m(\rho) = \lim_{n\to\infty} \frac{b_n}{1 + \sum_{i=0}^{n} a_i}$$

$$f_m(\rho) = \begin{cases} 0 & \text{if } v_{err} \text{ is not in } \rho \\ \dfrac{1}{\text{number of faults before } v_{err}} & \text{otherwise} \end{cases}$$

Quantitative Masking Game

The masking distance is defined by the value of the game:

$$\begin{aligned}\delta_m(A, A') \stackrel{\text{def}}{=} \text{val}(\mathcal{Q}_{A,A'}) &= \inf_{\pi_V \in \Pi_V} \sup_{\pi_R \in \Pi_R} f_m(\text{out}(\pi_R, \pi_V)) \\ &= \sup_{\pi_R \in \Pi_R} \inf_{\pi_V \in \Pi_V} f_m(\text{out}(\pi_R, \pi_V))\end{aligned}$$

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Quantitative Masking Game

The masking distance is defined by the value of the game:

$$\delta_m(A, A') \stackrel{\text{def}}{=} \text{val}(\mathcal{Q}_{A,A'}) = \inf_{\pi_V \in \Pi_V} \sup_{\pi_R \in \Pi_R} f_m(\text{out}(\pi_R, \pi_V))$$

$$= \sup_{\pi_R \in \Pi_R} \inf_{\pi_V \in \Pi_V} f_m(\text{out}(\pi_R, \pi_V))$$

this equality is guaranteed by a theorem

Theorem: $\delta_m(A, A') = 0$ iff $A \preceq_m A'$

Quantitative Masking Game (algorithm)

Definition 3.9. Given a strong masking game graph $\mathcal{G}_{A,A'}$, the sets U_i^j (for $i, j \geq 0$) are defined as follows:

$$U_i^0 = U_0^j = \emptyset,$$

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$$U_{i+1}^{j+1} = \{v' \mid v' \in V_{\mathrm{R}} \wedge \mathrm{post}(v') \cap U_{i+1}^{j} \neq \emptyset\}$$

$$\cup \{v' \mid v' \in V_{\mathrm{V}} \wedge \mathrm{post}(v') \subseteq \bigcup_{i' \leq i+1, j' \leq j} U_{i'}^{j'} \wedge \mathrm{post}(v') \cap U_{i+1}^{j} \neq \emptyset \wedge \mathrm{pr}_1(v') \notin \mathcal{F}\}$$

$$\cup \{v' \mid v' \in V_{\mathrm{V}} \wedge \mathrm{post}(v') \subseteq \bigcup_{i' \leq i, j' \leq j} U_{i'}^{j'} \wedge \mathrm{post}(v') \cap U_{i}^{j} \neq \emptyset \wedge \mathrm{pr}_1(v') \in \mathcal{F}\}$$

Furthermore, $U^k = \bigcup_{i \geq 0} U_i^k$ and $U = \bigcup_{k \geq 0} U^k$.

Quantitative Masking Game (algorithm)

Definition 3.9. Given a strong masking game graph $\mathcal{G}_{A,A'}$, the sets U_i^j (for $i, j \geq 0$) are defined as follows:

indicates that the error state is reached after at most *i*-1 faults

$$U_i^0 = U_0^j = \emptyset,$$

$$U_1^1 = \{v_{err}\},$$

$$U_{i+1}^{j+1} = \{v' \mid v' \in V_{\mathrm{R}} \wedge \mathrm{post}(v') \cap U_{i+1}^j \neq \emptyset\}$$
$$\cup \{v' \mid v' \in V_{\mathrm{V}} \wedge \mathrm{post}(v') \subseteq \textstyle\bigcup_{i' \leq i+1, j' \leq j} U_{i'}^{j'} \wedge \mathrm{post}(v') \cap U_{i+1}^j \neq \emptyset \wedge \mathrm{pr}_1(v') \notin \mathcal{F}\}$$
$$\cup \{v' \mid v' \in V_{\mathrm{V}} \wedge \mathrm{post}(v') \subseteq \textstyle\bigcup_{i' \leq i, j' \leq j} U_{i'}^{j'} \wedge \mathrm{post}(v') \cap U_i^j \neq \emptyset \wedge \mathrm{pr}_1(v') \in \mathcal{F}\}$$

Furthermore, $U^k = \bigcup_{i \geq 0} U_i^k$ and $U = \bigcup_{k \geq 0} U^k$.

Quantitative Masking Game (algorithm)

Definition 3.9. Given a strong masking game graph $\mathcal{G}_{A,A'}$, the sets U_i^j (for $i, j \geq 0$) are defined as follows:

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$$U_1^1 = \{v_{err}\},$$

$$U_{i+1}^{j+1} = \{v' \mid v' \in V_R \wedge \text{post}(v') \cap U_{i+1}^j \neq \emptyset\}$$

$$\cup \{v' \mid v' \in V_V \wedge \text{post}(v') \subseteq \textstyle\bigcup_{i' \leq i+1, j' \leq j} U_{i'}^{j'} \wedge \text{post}(v') \cap U_{i+1}^j \neq \emptyset \wedge \text{pr}_1(v') \notin \mathcal{F}\}$$

$$\cup \{v' \mid v' \in V_V \wedge \text{post}(v') \subseteq \textstyle\bigcup_{i' \leq i, j' \leq j} U_{i'}^{j'} \wedge \text{post}(v') \cap U_i^j \neq \emptyset \wedge \text{pr}_1(v') \in \mathcal{F}\}$$

Furthermore, $U^k = \bigcup_{i \geq 0} U_i^k$ and $U = \bigcup_{k \geq 0} U^k$.

indicates that the error state is reached after at most i-1 faults

Theorem:

$$\delta_m(A, A') = \begin{cases} \dfrac{1}{\min\{i \mid v_0^G \in U_i^j\}} & \text{if } v_0^G \in U \\ 0 & \text{otherwise} \end{cases}$$

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Everybody loves tables!

- Tool MaskD (developed by Luciano)
- Complexity (general):

  $\mathcal{O}(|E^G| * \log |V^G|)$

- Weak case requires reflexive-transitive construction, so add

  $\mathcal{O}(\max(|S|, |S'|)^{2.3727})$

- Complexity (deterministic)

  $\mathcal{O}(|E^G|)$

Shortest weighted path

Case Study	Redundancy	Masking Distance	Time	Time(Det)
Redundant Memory Cell	3 bits	0.333	0.7s	0.6s
	5 bits	0.25	2.5s	1.9s
	7 bits	0.2	7.2s	5.7s
	9 bits	0.167	1m.4s	1m11s
	11 bits	0.143	28m27s	26m10s
N-Modular Redundancy	3 modules	0.333	0.6s	0.5s
	5 modules	0.25	1.2s	0.7s
	7 modules	0.2	5.6s	3.8s
	9 modules	0.167	2m55s	2m32s
	11 modules	0.143	75m17s	72m48s
Dining Philosophers	2 phils	0.5	0.6s	0.6s
	3 phils	0.333	1.9s	0.9s
	4 phils	0.25	5.9s	2.6s
	5 phils	0.2	25.3s	24.1s
	6 phils	0.167	19m.23s	11m39s
Byzantine Generals	3 generals	0.5	0.9s	–
	4 generals	0.333	17.1s	–
	5 generals	0.333	429m54s	–
Raft LRCC (5)	1 follower	0	0.7s	0.8s
	2 followers	0	5.6s	3.6s
	3 followers	0	49m.50s	37m.53s
BRP(1)	1 retransm.	0.333	0.7s	–
	5 retransm.	0.143	0.8s	–
	10 retransm.	0.083	1.3s	–
	20 retransm.	0.045	3.9s	–
	40 retransm.	0.024	4.8s	–
BRP(5)	1 retransm.	0.333	4.2s	–
	5 retransm.	0.143	4.8s	–
	10 retransm.	0.083	6.1s	–
	20 retransm.	0.045	8.7s	–
	40 retransm.	0.024	18.6s	–
BRP(10)	1 retransm.	0.333	4.7s	–
	5 retransm.	0.143	6.4s	–
	10 retransm.	0.083	10.1s	–
	20 retransm.	0.045	20.5s	–
	40 retransm.	0.024	1m.9s	–

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Measuring Masking Fault-Tolerance

Pablo F. Castro, Pedro R. D'Argenio, Ramiro Demasi, Luciano Putruele

Dependable Systems dTime Talks
October, 2020