MSC 2020. *Primary*: 03E15; *Secondary*: 68Q15.

The first half of the paper studies the definability of certain classes of oracles and complexity classes. The authors show that the set of oracles **O** which separate **P** from **NP** is a lightface Π^0_2 subset of the Cantor space 2^ω; the improvement over the obvious Σ^0_3 definition is achieved by using a universal nondeterministic Turing machine (TM). **O** is also shown to be boldface $\boldsymbol{\Pi}^0_2$-hard (and hence complete): A continuous reduction of the set $\{b \in 2^\omega : b(n) = 1 \text{ for infinitely many } n\}$ to **O** is provided, by constructing an oracle A by layers, in such a way that for every 0 term in b, a layer of a fixed **EXP**-complete language is added to A, and for every 1 term, a family of **P** computations is diagonalized out, in a way similar to [T. Baker, J. Gill, and R. Solovay, SIAM J. Comput. **4** (1975) no. 4, 431–442. MR0395311].

The authors then review *quantum* TMs and obtain in an analogous way that the set of oracles **Q** separating **NP** from **BPQ** (which is the set of languages recognizable in polynomial time and with probability $\frac{2}{3}$ by a quantum TM) is boldface $\boldsymbol{\Pi}^0_2$-complete. It is later shown that the set of oracles **S** separating the polynomial hierarchy **PH** from **PSPACE** is $\boldsymbol{\Pi}^0_2$-complete as well.

Finally, **P** is proved to be $\boldsymbol{\Sigma}^0_2$-complete; the authors state that the method extends to many other familiar complexity classes going from **L** to **EXP**.

The second half of the paper begins with a review of infinite dimensional Ramsey theory and the Ellentuck topology on space of infinite subsets of the natural numbers. After this, the authors show that **O** is Ramsey-positive (actually showing that the set of oracles separating **NP** from **coNP** is). The argument is later adjusted to obtain the same result for **Q**.

The authors also show that the set of oracles A which satisfy $\mathbf{PSPACE}^A \subseteq \mathbf{IP}^A$ is Ramsey-large: This is obtained by thinning out any oracle to a *tame* one (which has at most one string of each length); the inclusion holds for all of those.

This last result favors the Ramsey perspective over the measure-theoretic one, since the set of all such A has measure zero [Chang, Richard; Chor, Benny; Goldreich, Oded; et al. The random oracle hypothesis is false, J. Comput. System Sci. **49** (1994), no. 1, 24–39. MR1284589]. Other results in the same line show that if **O** or **Q** (respectively, **S**) were shown to Ramsey-large (-positive), then the unrelativized separations would hold; the concept of (*very*) tame oracle is useful here too.