From Semantics to Spatial Distribution *

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Abstract. This work studies the notion of locality in the context of process specification. It relates naturally with other works where information about the localities of a program is obtained information from its description written down in a programming language.
This paper presents a new approach for this problem. In our case, the information about the system will be given in semantic terms using asynchronous transition systems. Given an asynchronous transition system we build an algebra of localities whose models are possible implementations of the known system. We present different results concerning the models for the algebra of localities. In addition, our approach neatly considers the relation of localities and non-determinism.

1 Introduction

In the framework of the so called true concurrency, the idea of causality has been widely studied [13,12,8,7,15]. Localities, an idea somehow orthogonal to causality, has become also interesting [1,4,5,10,11,9,3]. Causality states which events are necessary for the execution of a new one, while localities observe in which way the events are distributed. Both approaches have been shown not to be equivalent or to coincide in a very discriminating point [6,17].

The idea of the work on localities is to state where an event occurs given the already known structure of a process. Thus, the starting point is a process written in a clearly defined syntax. For instance, consider the process

$$a.c.\mathbf{stop} \;||_c\; c.b.\mathbf{stop} \tag{1}$$

where $||_c$ is the CSP parallel composition: there are two processes running together, but they must synchronize in the action c. This process may execute actions $a@\bullet|\emptyset$, $b@\emptyset|\bullet$, and $c@\bullet|\bullet$. The term in the right hand side of the @ indicates the places in which the action on the left side of @ occurs. In particular, the $\bullet$ shows in which side of the parallel operation the action takes place. Notice that a and b do not share any locality: a occurs at the left hand side of the

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parallel composition while b occurs at the right hand side. On the other hand, the process

$$a.c.b.\mathbf{stop} \tag{2}$$

presents the same sequence of actions a, c, and b, although in this case they occur exactly in the same place.

Besides, these works on localities have a nagging drawback: in some cases where non deterministic choice and parallel composition are involved, localities for actions do not seem to match our intuition. For instance, in the process

$$(a.\mathbf{stop} \mid\mid b.\mathbf{stop}) + (c.\mathbf{stop} \mid\mid d.\mathbf{stop}) \tag{3}$$

we have that $a@\bullet|\emptyset$ and $d@\emptyset|\bullet$. We could think that a and d do not share any resource, but in a causal-based model they are clearly in conflict: the occurrence of one of them forbids the occurrence of the other. From a causal point of view actions a and d must be sharing some locality.

The approach we chose is to deduce the distribution of events from the semantics of a given process. We use asynchronous transition systems [14,16] (ATS for short) to describe its behavior. Thus, in our case the architecture (i.e., the syntax) of the process is not known.

Our contribution consists of the statement and exploration of this original semantic-based approach. For each ATS we define an algebra of localities with a binary operation $\wedge$ that returns the common places of two events, and a constant 0 meaning "nowhere". The axioms of this algebra will give the minimal requirements needed for events to share or not to share some place. The axiomatization does not specify anything if such a statement cannot be deduced from the behavior. Thus, given the interpretation of the processes (1) and (2) we may deduce that a and c must have some common place, and we will write $a \wedge c \neq 0$.However, the axiomatization is not going to state whether $a \wedge b = 0$ or $a \wedge b \neq 0$. This will depend on the model chosen for the axiomatization, that gives the definitive criterion for the distribution of events: our models will be true implementations of ATS. We will show that our approach detects situations like the one described in process (3). In this case, we will have an explicit axiom saying that a and d share some common place, i.e, $a \wedge d \neq 0$.

In addition, we discuss different models for the algebra of localities of a given ATS. These models may be associated to a program whose specification was given in terms of the original ATS. First we introduce the *non-independence models* which consider whether two events are independent in the corresponding ATS. Then, we define models which take into account whether two events are adjacent.

Consider two events sharing a locality in a model $\mathcal{M}$ for a given ATS. If they share some locality in every possible model for this ATS, we call $\mathcal{M}$ a *minimal sharing model*. On the other hand, if two events share a locality in $\mathcal{M}$ only when they share a locality in any other model, then we call $\mathcal{M}$ a *maximal sharing model*. We show that the models concerning adjacency introduced in this work hold one of these properties.

The paper is organized as follows. Section 2 recalls the definition of ATS as well as some notions of graph theory. Section 3 introduces the algebra of localities. Six models for this algebra are presented in Section 4. Finally, conclusions and future works are given in Section 5.

2 Preliminaries

Asynchronous Transitions Systems Asynchronous transition systems [14,16] are a generalization of labeled transition systems. In ATSs, transitions are labeled with *events*, and each event represents a particular occurrence of an action. In addition, ATSs incorporate the idea of *independent* events. Two independent events can be executed in parallel, and so they cannot have resources in common. Formally, we define:

Definition 1. *Let* $\mathbf{A} = \{\alpha, \beta, \gamma, \ldots\}$ *be a set of* actions*. An* asynchronous transition system *is a structure* $T = (S, E, I, \longrightarrow, \ell)$ *where*

- $S = \{s, t, s', \ldots\}$ *is a set of* states *and* $E = \{a, b, c, \ldots\}$ *is a set of* events*;*
- $I \subseteq E \times E$ *is an irreflexive and symmetric relation of* independence*. We write* aIb *instead of* $(a, b) \in I$*;*
- $\longrightarrow \subseteq S \times E \times S$ *is the transition relation. We write* $s \xrightarrow{a} s'$ *instead of* $(s, a, s') \in \longrightarrow$*;*
- $\ell : E \to \mathbf{A}$ *is the* labeling function*.*

In addition, T has to satisfy the following axioms,

Determinism:	$s \xrightarrow{a} s' \wedge s \xrightarrow{a} s'' \implies s' = s''$
Forward stability:	$aIb \wedge s \xrightarrow{a} s' \wedge s \xrightarrow{b} s'' \implies \exists t \in E.\ s' \xrightarrow{b} t \wedge s'' \xrightarrow{a} t$
Commutativity:	$aIb \wedge s \xrightarrow{a} s' \wedge s' \xrightarrow{b} t \implies \exists s'' \in E.\ s \xrightarrow{b} s'' \wedge s'' \xrightarrow{a} t$

□

Example 1. In the Introduction we have mentioned a couple of examples. We are going to use them as running examples. To simplify notation, we use the same name for events and actions.

We can represent both $a.c.b.\mathbf{stop}$ and $a.c.\mathbf{stop}\ ||_c\ c.b.\mathbf{stop}$ by the ATS in Figure 1. Notice that for the second process, we could have aIb although that is not actually relevant. However, it is important to notice that $\neg(aIc)$ and $\neg(bIc)$ in both cases.

The ATS for process $(a.\mathbf{stop}\ ||\ b.\mathbf{stop}) + (c.\mathbf{stop}\ ||\ d.\mathbf{stop})$ is depicted in Figure 2. Notice that aIb and cId while any other pair of events is not independent. Shadowing is used to show the independence relation between events. □

Graphs A graph G consists of a finite set V of vertices together with a set X of unordered pairs of distinct vertices of V. The elements of X are the edges of G. We will note $\{v, w\} \in X$ as vw. We will write (V, X) for the graph G. Two vertices v and w are adjacent in G if $vw \in X$. Two edges e and f are adjacent if $e \cap f \neq \emptyset$.

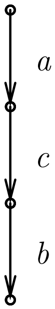

Fig. 1. The ATS for $a.c.b.\mathbf{stop}$ and $a.c.\mathbf{stop} \mid\mid_c c.b.\mathbf{stop}$

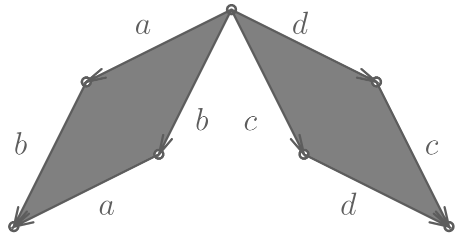

Fig. 2. The ATS for $(a.\mathbf{stop} \mid\mid b.\mathbf{stop}) + (c.\mathbf{stop} \mid\mid d.\mathbf{stop})$

Definition 2 (Subgraphs). *We call $H = (V', X')$ a subgraph of $G = (V, X)$, and note $H \subseteq G$, whenever $V' \subseteq V$ and $X' \subseteq X$. We write $\wp G$ for the set of all subgraphs of G. We write $\wp\wp G$ for the power set of $\wp G$.* □

A clique of a graph G is a maximal complete subgraph of G. As a complete graph is defined by its vertices, we will identify a clique with its corresponding set of vertices. We write $K(G)$ for the set of cliques of the graph G.

Lemma 1. *Let v and w be two vertices of $G = (V, X)$. Then, $vw \in X$ iff there exists a clique $K \in K(G)$ such that $vw \in X(K)$.*

3 The Algebra of Localities

In this section we explain how to obtain an *algebra of localities* from a given ATS. The algebra of localities is constructed over a semilattice by adding some particular axioms for each ATS.

Definition 3. *A* semilattice *is a structure $(\mathcal{L}, \wedge, 0)$ where $\wedge : \mathcal{L} \times \mathcal{L} \to \mathcal{L}$ and $0 \in \mathcal{L}$ satisfying the following axioms:*

$a \wedge b = b \wedge a$	*(commutativity)*	$a \wedge (b \wedge c) = (a \wedge b) \wedge c$	*(associativity)*
$a \wedge a = a$	*(idempotence)*	$a \wedge 0 = 0$	*(absorption)*

□

Each element in the set $\mathcal{L}$ refers to a set of *"places"*. In particular, 0 means *"nowhere"*. The operation $\wedge$ gives the *"common places"* between the operands. The axioms make sense under this new nomenclature. Commutativity says that the common places of a and b are the same as the common places of b and a. Associativity says that the common places of a, b, and c are always the same regardless we consider first the common places of a and b, or the common places of b and c. According to idempotency, the common places of a and itself are again the places of a. Finally, absorption says that any element of $\mathcal{L}$ has no common place with nowhere.

Now we introduce the concept of adjacent events. Two events are adjacent if they label two consecutive transitions, or two outgoing transitions from the same state.

Definition 4. *Let* $T = (S, E, I, \longrightarrow, \ell)$ *be an* ATS*. Two events* $a, b \in E$ *are* adjacent in T*, notation* $adj(a, b)$*, if and only if there exist* $s, s', s'' \in S$ *such that*

$$s \xrightarrow{a} s' \xrightarrow{b} s'' \quad or \quad s \xrightarrow{b} s' \xrightarrow{a} s'' \quad or \quad s \xrightarrow{a} s' \text{ and } s \xrightarrow{b} s''$$

□

We are interested in independence relation between adjacent events. When two events are not adjacent an observer cannot differentiate whether they are independent. For instance, in the ATS of Figure 1 it is not relevant whether a and b are independent since that does not affect the overall behavior.

The carrier set of the algebra of localities associated to an ATS includes an appropriate interpretation of its events. Such an interpretation refers to "the places where an event happens".

Definition 5. *Let* $T = (S, E, I, \longrightarrow, \ell)$ *be an* ATS*. The* algebra of localities associated to T *is a structure* $\mathcal{A} = (\mathcal{L}, E, \wedge, 0)$ *satisfying:*

1. $E \subseteq \mathcal{L}$*, and* $(\mathcal{L}, \wedge, 0)$ *is a semilattice*
2. aIb *and* $adj(a, b) \implies a \wedge b = 0$
3. $\neg(aIb)$ *and* $adj(a, b) \implies a \wedge b \neq 0$

□

Example 2. For the ATS of Figure 1 we obtain the following axioms:

$$a \wedge c \neq 0 \qquad\qquad c \wedge b \neq 0$$

Notice that the axiom system does not say whether $a \wedge b \neq 0$ or $a \wedge b = 0$. Thus, the algebra does not contradict the decision of implementing the ATS either with process $a.c.b.$**stop**, in which a and b occur in the same place, or with $a.c.$**stop** $||_c$ $c.b.$**stop**, in which a and b occur in different places.

For the ATS of Figure 2 we obtain the following axioms:

$$\begin{array}{ccc} a \wedge b = 0 & a \wedge c \neq 0 & b \wedge c \neq 0 \\ c \wedge d = 0 & a \wedge d \neq 0 & b \wedge d \neq 0 \end{array}$$

Notice that the axioms state that a and d must share some places. On the other hand, as we already said, other approaches to localities cannot identify such a conflict. □

4 Models for the Algebra of Localities

In this section we introduce several models for the algebra of localities associated to a given ATS, thus proving its soundness. Each of our models may be an implementation.The interpretation for the events will be based on the relations of independence and adjacency. The names of the models are taken from these basic relations.

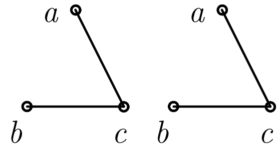

Fig. 3. I models for $a.c.b.$**stop** and $a.c.$**stop** $||_c$ $c.b.$**stop**

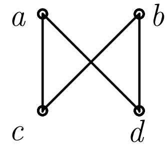

Fig. 4. I model for ($a.$**stop** || $b.$**stop**) + ($c.$**stop** || $d.$**stop**)

The I Models The non-independence models (I models for short) for the algebra of localities associated to a given ATS assign common places to non-independent events. We define the non-independent models I and I2, based on cliques and edges respectively.

Let $T = (S, E, I, \longrightarrow, \ell)$ be an ATS. We define the graph $G^I = (E, \{\{a,b\} \subseteq E \mid \neg(aIb)\})$. We define the interpretation of an event a in the model I (I2) to be the set of cliques (edges) in G^I in which a appears.

$$[\![a]\!]^I \stackrel{\text{def}}{=} \{A \in K(G^I) \mid a \in A\} \qquad\qquad ([\![a]\!]^{I2} \stackrel{\text{def}}{=} \{A \in X(G^I) \mid a \in A\})$$

Each set $A \in [\![a]\!]^I$ is a different place where a may happen: each place is identified with the set of all events that can happen there. Moreover, an event can happen in several places simultaneously. The operation $\wedge$ of the algebra of localities is interpreted as the intersection $\cap$ between sets, and the constant 0 is interpreted as the empty set $\emptyset$.

Example 3. For the ATS in Figure 1 with aIb, we obtain the graph G^I on the left of Figure 3. This implementation uses two places or localities. One of them is shared by a and c, and the other by b and c. So, this model is well suited for the implementation $a.c.$**stop** $||_c$ $c.b.$**stop**. In this case, both I and I2 interpretations coincide. These could be written down as

$$[\![a]\!]^I = \{\{a,c\}\} \qquad [\![b]\!]^I = \{\{b,c\}\} \qquad [\![c]\!]^I = \{\{a,c\},\{b,c\}\}$$

We have a new interpretation in case a and b are not independent. We can see it on the right of the Figure 3. Now, every event occurs in the same place. In other words, if $\neg(aIb)$, the I model implements $a.c.b.$**stop**.

$$[\![a]\!]^I = [\![b]\!]^I = [\![c]\!]^I = \{\{a,b,c\}\}$$

A different interpretation is established for model I2. In this case, we have

$$[\![a]\!]^{I2} = \{\{a,c\},\{a,b\}\} \quad [\![b]\!]^{I2} = \{\{b,c\},\{a,b\}\} \quad [\![c]\!]^{I2} = \{\{a,c\},\{b,c\}\}$$

This model implements the program $a.b.$**stop** || $a.c.$**stop** || $c.b.$**stop** that uses three localities.

For the ATS of Figure 2 we have G^I depicted in Figure 4. The execution of a requires two places, one shared with c and the other with d. Thus, the event

a prevents the execution of d by occupying a place required by this event. This reflects the fact that selection between non independent events occurs actually in a place. For this implementation, we have

$$[\![a]\!]^I = \{\{a,c\},\{a,d\}\} \qquad [\![b]\!]^I = \{\{b,c\},\{b,d\}\}$$
$$[\![c]\!]^I = \{\{a,c\},\{b,c\}\} \qquad [\![d]\!]^I = \{\{a,d\},\{b,d\}\}$$

□

Now we prove that non-independence models are indeed models for the algebra of localities.

Theorem 1 (Soundness). *Let* $T = (S, E, I, \longrightarrow, \ell)$ *be an* ATS*,* $\mathcal{A}$ *its algebra of localities, and* $[\![E]\!]^{I(I2)} = \{[\![a]\!]^{I(I2)} \mid a \in E\}$*. Then,*

$$\mathcal{M}^I \stackrel{\text{def}}{=} \left(\wp\wp\left(G^I\right), [\![E]\!]^I, \cap, \emptyset\right) \text{ and } \mathcal{M}^{I2} \stackrel{\text{def}}{=} \left(\wp\wp\left(G^I\right), [\![E]\!]^{I2}, \cap, \emptyset\right)$$

are models for $\mathcal{A}$*.*

Proof. By definition, $[\![E]\!]^{I2} \subseteq \wp\wp\left(G^I\right)$. Moreover,$\left(\wp\wp\left(G^I\right), \cap, \emptyset\right)$ is a well known semilattice.

Suppose that aIb and $adj(a,b)$. They are not adjacent in G^I, and so there is no edge between a and b in G^{I2}. Thus, $[\![a]\!]^{I2} \cap [\![b]\!]^{I2} = \emptyset$.

Finally, suppose that $\neg(aIb)$ and $adj(a,b)$. Then, $ab \in X(G^I)$, and hence $[\![a]\!]^{I2} \cap [\![b]\!]^{I2} \neq \emptyset$.

The proof for model I is similar, taking into account Lemma 1. □

We can see that, although localities may change, the relation between these two models remain substantially unchanged. More explicitly, two events sharing resources in any of these models will share resources in the other.

Theorem 2. $\mathcal{M}^I \models a \wedge b \neq 0$ *if and only if* $\mathcal{M}^{I2} \models a \wedge b \neq 0$

Minimal Sharing Models: IJ and IJ2 In the models IJ and IJ2 we assign common places to events that are both adjacent and non-independent. We will show they are minimal sharing in the following sense : whenever two events share a place for this models, they will share a place in any other model.

Let $T = (S, E, I, \longrightarrow, \ell)$ be an ATS. Taking adjacent events into account we define the graph $G^{IJ} = (E, \{\{a,b\} \subseteq E \mid \neg(aIb) \text{ and } adj(a,b)\})$. As before, we define the interpretation of an event a to be the set of cliques or edges in G^{IJ} where a appears.

$$[\![a]\!]^{IJ} \stackrel{\text{def}}{=} \{A \in K(G^{IJ}) \mid a \in A\} \qquad [\![a]\!]^{IJ2} \stackrel{\text{def}}{=} \{A \in X(G^{IJ}) \mid a \in A\}$$

Theorem 3 (Soundness). *Let* $T = (S, E, I, \longrightarrow, \ell)$ *be an* ATS *and let* $\mathcal{A}$ *be its algebra of localities. Then,*

$$\mathcal{M}^{IJ} \stackrel{\text{def}}{=} \left(\wp\wp\left(G^{IJ}\right), [\![E]\!]^{IJ}, \cap, \emptyset\right) \text{ and } \mathcal{M}^{IJ2} \stackrel{\text{def}}{=} \left(\wp\wp\left(G^{IJ}\right), [\![E]\!]^{IJ2}, \cap, \emptyset\right)$$

are models for $\mathcal{A}$*.*

Theorem 4. $\mathcal{M}^{IJ} \models a \wedge b \neq 0$ *if and only if* $\mathcal{M}^{IJ2} \models a \wedge b \neq 0$

These models enjoy the following property: if two events are distributed (i.e., do not share a place) in some model for the algebra of localities of a given ATS, they are also distributed in these models. This justifies calling them minimal sharing models. The following theorem states the counter positive of that property.

Theorem 5. *Let* $T = (S, E, I, \longrightarrow, \ell)$ *be an* ATS *and let* $\mathcal{A}$ *be its algebra of localities. Let* $\mathcal{M}$ *be any model for* $\mathcal{A}$. *Then, for all events* $a, b \in E$,

$$\mathcal{M}^{IJ2} \models a \wedge b \neq 0 \implies \mathcal{M} \models a \wedge b \neq 0$$

Proof. Suppose $\mathcal{M}^{IJ2} \models a \wedge b \neq 0$, that is $[\![a]\!]^{IJ2} \cap [\![b]\!]^{IJ2} \neq \emptyset$. Thus $ab \in X(G^{IJ})$, which implies $\neg(aIb)$ and $adj(a, b)$. So, by Definition 5, $\mathcal{A} \vdash a \wedge b \neq 0$. Hence, for any model $\mathcal{M}$ of $\mathcal{A}$, $\mathcal{M} \models a \wedge b \neq 0$. □

An easy application of Theorem 4 give us this corollary:

Corollary 1. $\mathcal{M}^{IJ}$ *is a minimal sharing model.* □

Maximal Sharing Models: InJ and InJ2 In a similar way we construct a model of maximal sharing. In this case, two events share places unless they must execute independently. We call them InJ models because they may require non adjacency.

Let $T = (S, E, I, \longrightarrow, \ell)$ be an ATS. We define the graph $G^{InJ} = (E, \{\{a, b\} \subseteq E \mid \neg\ (\ aIb$ and $adj(a, b)\ )\})$. We define the interpretation of an event a to be the set of cliques or edges in G^{InJ} where a appears.

$$[\![a]\!]^{InJ} \stackrel{\text{def}}{=} \{A \in K(G^{InJ}) \mid a \in A\} \qquad [\![a]\!]^{InJ2} \stackrel{\text{def}}{=} \{A \in X(G^{InJ}) \mid a \in A\}$$

Theorem 6 (Soundness). *Let* $T = (S, E, I, \longrightarrow, \ell)$ *be an* ATS *and let* $\mathcal{A}$ *be its algebra of localities. Then,*

$$\mathcal{M}^{InJ} \stackrel{\text{def}}{=} \left(\wp\wp\left(G^{InJ}\right), [\![E]\!]^{InJ}, \cap, \emptyset\right) \quad \textit{and}$$
$$\mathcal{M}^{InJ2} \stackrel{\text{def}}{=} \left(\wp\wp\left(G^{InJ}\right), [\![E]\!]^{InJ2}, \cap, \emptyset\right)$$

are models for $\mathcal{A}$.

Theorem 7. $\mathcal{M}^{InJ2} \models a \wedge b \neq 0$ *if and only if* $\mathcal{M}^{InJ} \models a \wedge b \neq 0$

This model describes maximal sharing in the sense that if two events are distributed in it, they are distributed in any other model. The following theorems state this property for the InJ models.

Theorem 8. *Let* $T = (S, E, I, \longrightarrow, \ell)$ *be an* ATS *and let* $\mathcal{A}$ *be its algebra of localities. Let* $\mathcal{M}$ *be any model for* $\mathcal{A}$. *Then, for all events* $a, b \in E$,

$$\mathcal{M}^{InJ2} \models a \wedge b = 0 \implies \mathcal{M} \models a \wedge b = 0$$

Corollary 2. *$\mathcal{M}^{InJ}$ is a maximal sharing model.* □

Example 4. We can see on the right of Figure 3 the graph G^{InJ} for the ATS of Figure 1, no matter whether a and b are independent. Thus, we obtain the following interpretation in the maximal sharing model.

$$[\![a]\!]^{InJ} = [\![b]\!]^{InJ} = [\![c]\!]^{InJ} = \{\{a, b, c\}\}$$

Thus, $\mathcal{M}^{InJ} \models a \wedge b \neq 0$. However, from Example 3 we know that when aIb, $\mathcal{M}^{I} \models a \wedge b = 0$. So, we have that I models are not maximal sharing models.

We have that for the same ATS when a and b are not independent, $\mathcal{M}^{I} \models a \wedge b \neq 0$ and $\mathcal{M}^{IJ} \models a \wedge b = 0$. Thus, I models are not minimal sharing models either. □

5 Conclusions

In this work we have exploited the information about localities hidden in the ATS definition. Such information helps us to find implementations of systems with certain properties, like maximal or minimal sharing of localities.

The way to state how the locality of events are related is by means of the algebra of localities. We have introduced several models for this algebra and showed that this is not a trivial set of models. Figure 5 summarizes our result in Section 4. The up-going arrows in the picture mean that sharing on the lower models implies sharing on the upper models.

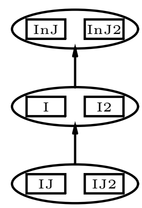

Fig. 5. Models of localities

We also have shown that our semantic approach exposes clearly difficulties arisen in syntactic language oriented approaches when dealing with non deterministic choices.

As a consequence of this work we can extract locality information from a specification written in terms of ATS. So, ATS formalism appears as a good candidate to become a theoretical assembler for distributed programming. At least, there are three interesting directions to continue this work. One of them is to go on a deeper comprehension of locality models. The nature of the hierarchy of models seems far away from being trivial, requiring more detailed studies on its structure. We believe that research in this direction will allow us to detect not only minimal sharing models, but also models with some constraints which require less localities to work.

We may develop the same strategy for other semantic formalisms, that is, to associate an algebra of localities and to obtain a model as before. Event structures [12], from where the notion of independence can be easily derived, would be a good candidate to study.

Another direction for future work would be to extend ATS with new characteristics. Time is a natural factor to consider in this extensions, as far as resources are used for events during certain time. A relation between a not yet defined timed ATS and timed graphs [2] would enable us to move into timed systems, where tools and methods for automatic verification have been developed.

Another way for continuing our work is the development of a toolkit for description of systems based in ATS. We believe that semantic studies in programming must come together with software development, and so implementation of good toolkits for both theoretical and practical developments will become more important in future.

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