The authors define a new exactness property of an arbitrary category C, *involution-rigidness*. For arrows f, g in C, let $g \mid f$ abbreviate that there exists h such that $f = gh$. Very succinctly, C has the involution-rigidness property if

$$m \mid g \text{ and } \forall p\,(ip = p \Rightarrow m \mid p) \text{ imply } m \mid ig, \tag{1}$$

for all arrows m, p, and an involution i, all with the same codomain. In the particular case of a variety of algebras $\mathcal{V}$, it is equivalent to the following: For every $X \in \mathcal{V}$ and an involution $i : X \to X$, every subalgebra $M \subseteq X$ containing the fixpoints of i is stable under i (i.e., $i(M) \subseteq M$).

In the general case, categories having the involution-rigidness property are (a) any additive category, (b) any preorder, (c) the dual of Set, and (d) the variety of commutative difference monoids (i.e., $(M, +, 0, d)$ with $d(x + y, y) = x$). Varieties of algebras having the property are Mal'cev, but the converse does not hold: The category of groups does not have the involution-rigidness property.

The authors give a simple Mal'cev condition characterizing varieties of algebras having this property; the proof of this uses the involution on the free algebra $F(x, y)$ determined by $i(x) = y$ and $i(y) = x$, and M its subalgebra generated by y and the fixpoints of i. This idea generalizes to any category having pullbacks and coproducts, for which to check the involution-rigidness property is enough to consider in (1) g to be the coproduct injection $\iota_1 : N \to N + N$ and i the involution $(\iota_2, \iota_1) : N + N \to N + N$ for every object N in the category.

The rest of the paper is devoted to a weak version of the involution-rigidness property.