The author works using *universal variants* of the systems of Bounded Arithmetic [S. R. Buss, First-order proof theory of arithmetic, in *Handbook of proof theory*, 79–147, Stud. Logic Found. Math., 137, North-Holland, Amsterdam. MR1640326]. These are defined as the standard T_2, T^1_2, S^1_2 but in a language containing a symbol for each poly-time function (using an oracle α), and supplemented with the theory $\forall\mathsf{PV}(\alpha)$ consisting of the universal sentences in that language that are valid in all models $(\mathbb{N}, \alpha^{\mathbb{N}})$ with $\alpha^{\mathbb{N}}$ ranging over subsets of $\mathbb{N}$.

The use of these variants allows a direct translation from poly-time reduction to logical deduction over the universal variant $\mathsf{S}^1_2(\mathsf{PV}(\alpha))$, and similarly for ℓ-query reduction and $\forall\mathsf{PV}(\alpha)$. After a review of (existential, unnested) *basic formulas* and unary and binary codes for structures, the author strengthens [S. R. Buss and A. S. Johnson, Propositional proofs and reductions between NP search problems, Ann. Pure Appl. Logic **163** (2012), no. 9, 1163–1182. MR2926277].

In a second part of the paper, the concepts of weak and strong principles, which involve partial structures, are defined. It is to be noted that although the value "undefined" ($\frac{1}{2}$) works as expected when it occurs in terms, for formulas it behaves as an intermediate truth value (above falsity and below truth). A principle valid in the finite (total structures) is *weak* if its validity still holds in a structure even if a fair part of it (at least a positive power of the cardinal of the universe) is undefined. The author notes that the (n^2 to n)-pigeonhole principle is the only weak principle available at the moment, but nevertheless independence results for non-weak principles can be obtained (weakness is not preserved by deduction over $\forall\mathsf{PV}(\alpha)$). The definition of *strong* principle is a bit technical and involves looking if its negation has an infinite model "almost having" finite substructures (with a "growth" bounded by an infinitesimal power of the cardinal of the candidate subuniverse). Section 4.4 provides a nice collection of examples of both behaviors.

Finally, the author applies the forcing machinery developed in [A. Atserias and M. Müller, Partially definable forcing and bounded arithmetic, Arch. Math. Logic **54** (2015), no. 1-2, 1–33. MR3304734] to *typical forcings*, in which the forcing poset P is a subset of the model M, there are formulas that encode its structure of P into M, and the standard properties of set-forcing concerning dense sets hold. Moreover, it is required that the forcing is *conservative*: it does not change truths not involving the oracle (name of the generic) α. As in [Atserias and Müller] above, the Definability of forcing is not granted, but it is one of the technical points of the paper.

This development allows to obtain extensions of models of $\forall\mathsf{PV}$ that satisfy $\mathsf{T}^1_2(\mathsf{PV}(\alpha))$ and then to prove the independence of strong principles from weak ones. In particular, Riis' finitization theorem [Making infinite structures finite in models of second order bounded arithmetic, in *Arithmetic, proof theory, and computational complexity (Prague, 1991)*, 289–319, Oxford Logic Guides, 23, Oxford Sci. Publ, Oxford Univ. Press, New York. MR1236468] follows as a corollary.

I suggest the reader to look at the first pages of the paper for an excellent summary of several details not considered in the present review.