

Two states p, q of a labelled transition system $(S, \Delta, \longrightarrow)$ are *bisimilar* (notation $p \sim q$) if there exists a *bisimulation* that relates them; that is, a relation R such that if $a \in \Delta$ is a label and $s R t$,

$$\begin{aligned} s \xrightarrow{a} s' &\implies \exists t' : s' R t' \wedge t \xrightarrow{a} t'; \\ t \xrightarrow{a} t' &\implies \exists s' : s' R t' \wedge s \xrightarrow{a} s'. \end{aligned}$$

Only one of these is necessary if one assumes R is symmetric. Since \sim is the largest bisimulation, to prove $s \sim t$ it is enough to find a symmetric R such that $s R t$ implies

$$s \xrightarrow{a} s' \implies \exists t' : s' \sim R \sim t' \wedge t \xrightarrow{a} t', \quad (1)$$

where juxtaposition indicates relational composition; we say that R *progresses* to $\sim R \sim$ when (1) holds. This technique to prove bisimilarity is known as *bisimulation up-to* \sim and was introduced by Milner [Calculi for synchrony and asynchrony. Theoret. Comput. Sci. 25 (1983), no. 3, 267–310. MR0716132]. Sangiorgi [On the bisimulation proof method. Math. Structures Comput. Sci. 8 (1998), no. 5, 447–479. MR1652718] studied variants of the set function $R \mapsto \sim R \sim$. It is apparent that the bigger the value of the function, the easier the proof of bisimilarity. Among the functions which lead to proofs of bisimilarity, Sangiorgi singled out the class of *respectful* functions (preserving the combination of inclusion and progression between relations), and proved that there is a maximal such function.

This paper presents an explicit characterization of the largest respectful function \mathcal{F} . It uses the ordinal stratification of bisimulation $\sim = \bigcap_{\alpha} \sim_{\alpha}$ (see [Robin Milner, Communication and Concurrency. Prentice-Hall International Series in Computer Science (1989)]), and essentially there are two cases. If $R \subseteq \sim$, then $\mathcal{F}(R) = \sim$. Otherwise, $\mathcal{F}(R) = \sim_{\alpha}$ for the largest α such that $R \subseteq \sim_{\alpha}$.

The authors finally compare this approach to analogous notions in the literature involving complete lattices.