Uncertainty-Based Semantics for Multi-Agent Knowing How Logics

Carlos Areces Raul Fervari Andrés R. Saravia

FAMAF, Universidad Nacional de Córdoba, & CONICET, Argentina

{carlos.areces,rfervari}@unc.edu.ar, andresrsaravia@mi.unc.edu.ar

Fernando R. Velázquez-Quesada

ILLC, Universiteit van Amsterdam, The Netherlands

F.R.VelazquezQuesada@uva.nl

We introduce a new semantics for a multi-agent epistemic operator of *knowing how*, based on an indistinguishability relation between plans. Our proposal is, arguably, closer to the standard presentation of *knowing that* modalities in classical epistemic logic. We study the relationship between this semantics and previous approaches, showing that our setting is general enough to capture them. We also define a sound and complete axiomatization, and investigate the computational complexity of its model checking and satisfiability problems.

1 Introduction

Epistemic logic (EL; [19, 9]) is a logical formalism tailored for reasoning about the knowledge of abstract autonomous entities, commonly called agents (e.g., a human being, a robot, a vehicle). Most standard epistemic logics deal with an agent's knowledge about the truth-value of propositions (the notion of *knowing that*). Thus, they focus on the study of sentences like *"the agent knows that it is sunny in Paris"* or *"the robot knows that it is standing next to a wall"*.

At the semantic level, EL formulas are typically interpreted over relational models [6, 5]: essentially, labeled directed graphs. The elements of the domain (called *states* or *worlds*) represent different possible situations. Each agent has associated a relation (interpreted as an *epistemic indistinguishability* relation), used to represent its uncertainty: related states are considered indistinguishable for the agent. An agent is said to know that a proposition φ is true at a given state s if and only if φ holds in all states she cannot distinguish from s. It is typically assumed that the indistinguishability relation is an equivalence relation. In spite of its simplicity, this indistinguishability-based representation of knowledge has several advantages. First, it also represents the agent's *high-order* knowledge (knowledge about her own knowledge and that of other agents). Second, it allows a very natural representation of actions through which knowledge changes (epistemic updates, see, e.g., [7, 4]).

In recent years, other patterns of knowledge besides knowing that have been investigated (see the discussion in [31]). Some examples are *knowing whether* [16, 10], *knowing why* [1, 33] and *knowing the value* [14, 2, 8]. Motivated by different scenarios in philosophy and AI, languages for reasoning about *knowing how* assertions [11] are particularly interesting. Intuitively, an agent

Submitted to:
TARK 2021

knows how to achieve φ given ψ if she has the *ability* to guarantee that φ will be the case whenever she is in a situation in which ψ holds.

There is a large literature connecting knowing how with logics of knowledge and action (see, e.g., [27, 28, 24, 20, 18]). However, these proposals for representing knowing how have been the target of criticisms. The main issue is that a simple combination of standard operators expressing *knowing that* and *ability* (see, e.g., [21]) does not seem to lead to a natural notion of knowing how (see [22, 17] for a discussion).

Taking these considerations into account, [30, 31, 32] introduced a framework based on a knowing how binary modality $\mathsf{Kh}(\psi,\varphi)$. At the semantic level, this language is also interpreted over relational models — called in this context labeled transition systems (LTSs). But relations do not represent indistinguishability anymore; they rather describe the actions an agent has at her disposal (in some sense, her *abilities*). An edge labeled a going from state w to state u indicates that the agent can execute action a to transform state w into u. In the proposed semantics, $\mathsf{Kh}(\psi,\varphi)$ holds if and only if there is a "plan" (a sequence of actions satisfying a constraint called strong executability) in the LTS that unerringly leads from every ψ-state only to φ-states. Other variants of this knowing how operator follow a similar approach (see [25, 26, 12, 29]).

In these proposals, relations are interpreted as the agent's available actions (as it is done in, e.g., propositional dynamic logic [15]); and the knowing how of an agent is directly defined by what these actions can achieve. This is in sharp contrast with EL, where relational models have two kinds of information: ontic facts about a given situation (represented by the current state in the model), and the particular perspective that agents have (represented by the possible states available in the model, and their respective indistinguishability relation between them).[1] If one would like to mirror the situation in EL, it seems natural that knowing how should be defined in terms of some kind of indistinguishability over the information provided by an LTS. Such an extended model would be able to capture both the abilities of an agent as given by her available actions, together with the (in)abilities that arise when considering two different actions/plans/executions indistinguishable.

This paper introduces a new semantics for $\mathsf{Kh}_i(\psi,\varphi)$, a multi-agent version of the knowing how modality. The crucial idea is the inclusion of a notion of epistemic indistinguishability over plans, in the spirit of the *strategy indistinguishability* of, e.g., [23, 3]. We interpret formulas over an *uncertainty-based LTS* (LTS$^{\mathsf{U}}$) which is an LTS equipped with an indistinguishability relation over plans. An agent may have different alternatives at her disposal to try to achieve a goal, all "as good as any other" (and in that sense indistinguishable) as far as she can tell. In this way, LTS$^{\mathsf{U}}$s aims to reintroduce the notion of epistemic indistinguishability, now at the level of plans. Moreover, the use of LTS$^{\mathsf{U}}$s leads to a natural definition of operators that represent dynamic aspects of knowing how (e.g., the concept of *learning how* can be modeled by eliminating uncertainty between plans).

Our contributions. They can be summarized as follows: ***(1)*** We introduce a new semantics for $\mathsf{Kh}_i(\psi,\varphi)$ (for i an agent) that reintroduces the notion of epistemic indistinguishability from classical EL. ***(2)*** We show that the logic obtained is weaker (and this is an advantage, as we will discuss) than the logic from [30, 31, 32]. Still, the new semantics is general enough to capture previous proposals by imposing adequate conditions on the class of models. ***(3)*** We present a sound and complete axiomatization for the logic over the class of all LTS$^{\mathsf{U}}$s. ***(4)*** We prove that

[1] Notice that in a multi-agent scenario, all agents share the same ontic information, and differ on their epistemic interpretation of it. We will come back to this later.

the satisfiability problem for the new logic is NP-complete, whereas model checking is in P.

Outline. Sec. 2 recalls the framework of [30, 31, 32], including its axiom system. Sec. 3 introduces *uncertainty-based LTS*, indicating how it can be used for interpreting a multi-agent version of the knowing how language, and providing an axiom system in Sec. 3.1. Sec. 3.2 studies the correspondence between our semantics and the ones in the previous proposals. Sec. 3.3 studies the computational complexity of model checking and the satisfiability problem for our logic. Sec. 4 provides conclusions and future lines of research.

2 A logic of knowing how

This section recalls the basic *knowing how* framework from [30, 31, 32].

Syntax and semantics. Throughout the text, let Prop be a countable non-empty set of propositional symbols.

Definition 2.1 Formulas of the language L_{Kh} are given by the following grammar:

$$\varphi ::= p \mid \neg\varphi \mid \varphi \vee \varphi \mid \mathsf{Kh}(\varphi, \varphi),$$

with $p \in \mathsf{Prop}$. Other Boolean connectives are defined as usual. Formulas of the form $\mathsf{Kh}(\psi, \varphi)$ are read as *"when ψ holds, the agent knows how to make φ true"*. ⊣

In [30, 31, 32] (and variations like [26, 25]), formulas of L_{Kh} are interpreted over *labeled transition systems*: relational models in which the relations describe the state-transitions available to the agent. Throughout the text, let Act be a denumerable set of (basic) action names.

Definition 2.2 (Actions and plans) Let Act^* be the set of finite sequences over Act. Elements of Act^* are called *plans*, with ϵ being the *empty plan*. Given $\sigma \in \mathsf{Act}^*$, let $|\sigma|$ be the length of σ ($|\epsilon| := 0$). For $0 \le k \le |\sigma|$, the plan σ_k is σ's initial segment up to (and including) the kth position (with $\sigma_0 := \epsilon$). For $0 < k \le |\sigma|$, the action $\sigma[k]$ is the one in σ's kth position. ⊣

Definition 2.3 (Labeled transition systems) A *labeled transition system (*LTS*)* for Prop and Act is a tuple $\mathcal{S} = \langle \mathrm{W}, \mathrm{R}, \mathrm{V} \rangle$ where W is a non-empty set of states (also denoted by $\mathrm{D}_{\mathcal{S}}$), $\mathrm{R} = \{\mathrm{R}_a \subseteq \mathrm{W} \times \mathrm{W} \mid a \in \mathsf{Act}\}$ is a collection of binary relations on W, and $\mathrm{V} : \mathrm{W} \to 2^{\mathsf{Prop}}$ is a labelling function. Given an LTS $\mathcal{S}$ and $w \in \mathrm{D}_{\mathcal{S}}$, the pair $(\mathcal{S}, w)$ is a *pointed* LTS (parentheses are usually dropped).⊣

An LTS describes the *abilities* of the agent; thus, sometimes (e.g., [30, 31, 32]) it is also called an *ability map*. Here are some useful definitions.

Definition 2.4 Let $\{\mathrm{R}_a \subseteq \mathrm{W} \times \mathrm{W} \mid a \in \mathsf{Act}\}$ be a collection of binary relations. Define $\mathrm{R}_\epsilon := \{(w, w) \mid w \in \mathrm{W}\}$ and, for $\epsilon \neq \sigma \in \mathsf{Act}^*$ and $a \in \mathsf{Act}$, $\mathrm{R}_{\sigma a} := \{(w, u) \in \mathrm{W} \times \mathrm{W} \mid \exists v \in \mathrm{W} \text{ s.t. } (w, v) \in \mathrm{R}_\sigma \text{ and } (v, u) \in \mathrm{R}_a\}$. Take a plan $\sigma \in \mathsf{Act}^*$: for $u \in \mathrm{W}$ define $\mathrm{R}_\sigma(u) := \{v \in \mathrm{W} \mid (u, v) \in \mathrm{R}_\sigma\}$, and for $U \subseteq \mathrm{W}$ define $\mathrm{R}_\sigma(U) := \bigcup_{u \in U} \mathrm{R}_\sigma(u)$. ⊣

Intuitively, [30, 31, 32] defines that an agent knows how to achieve φ given ψ when she has an appropriate plan that allows her to go from any situation in which ψ holds to only states in which φ holds. A crucial part is, then, what "appropriate" is taken to be.

Definition 2.5 (Strong executability) Let $\{\mathrm{R}_a \subseteq \mathrm{W} \times \mathrm{W} \mid a \in \mathsf{Act}\}$ be a collection of binary relations. A plan $\sigma \in \mathsf{Act}^*$ is *strongly executable* (SE) at $u \in \mathrm{W}$ if and only if $v \in \mathrm{R}_{\sigma_k}(u)$ implies $\mathrm{R}_{\sigma[k+1]}(v) \neq \emptyset$ for every $k \in [0..|\sigma| - 1]$. We define the set $\mathrm{SE}(\sigma) := \{w \in \mathrm{W} \mid \sigma \text{ is SE at } w\}$. ⊣

Thus, strong executability asks for *every* partial execution of the plan (which might be ϵ) to be completed. With this notion, formulas in $\mathsf{L_{Kh}}$ are interpreted over an LTS as follows (the semantic clause for the Kh modality is equivalent to the one found in the original papers).

Definition 2.6 ($\mathsf{L_{Kh}}$ over LTSs) The relation $\models$ between a pointed LTS $\mathcal{S},w$ (with $\mathcal{S} = \langle \mathrm{W}, \mathrm{R}, \mathrm{V}\rangle$ an LTS over Act and Prop) and formulas in $\mathsf{L_{Kh}}$ (over Prop) is defined inductively as follows:

$$\begin{array}{lll}
\mathcal{S},w \models p & \mathit{iff}_{def} & w \in \mathrm{V}(p), \\
\mathcal{S},w \models \neg\varphi & \mathit{iff}_{def} & \mathcal{S},w \not\models \varphi, \\
\mathcal{S},w \models \varphi \vee \psi & \mathit{iff}_{def} & \mathcal{S},w \models \varphi \text{ or } \mathcal{S},w \models \psi, \\
\mathcal{S},w \models \mathsf{Kh}(\psi,\varphi) & \mathit{iff}_{def} & \exists\sigma \in \mathsf{Act}^* \text{ such that } \textbf{(Kh-}\mathit{1}\textbf{)}\ [\![\psi]\!]^{\mathcal{S}} \subseteq \mathrm{SE}(\sigma) \text{ and } \textbf{(Kh-}\mathit{2}\textbf{)}\ \mathrm{R}_\sigma([\![\psi]\!]^{\mathcal{S}}) \subseteq [\![\varphi]\!]^{\mathcal{S}},
\end{array}$$

with $[\![\varphi]\!]^{\mathcal{S}} := \{w \in \mathrm{W} \mid \mathcal{S},w \models \varphi\}$ (the elements of $[\![\varphi]\!]^{\mathcal{S}}$ are sometimes called φ-states). ⊣

$\mathsf{Kh}(\psi,\varphi)$ holds when there is a plan σ such that, when it is executed at any ψ-state, it will always complete every partial execution (condition **(Kh-*1*)**), ending unerringly in states satisfying φ (condition **(Kh-*2*)**). Notice that Kh acts *globally*, i.e., $[\![\mathsf{Kh}(\psi,\varphi)]\!]^{\mathcal{S}}$ is either $\mathrm{D}_{\mathcal{S}}$ or $\varnothing$.

Axiomatization. The universal modality [13], interpreted as truth in every state of the model, is definable in $\mathsf{L_{Kh}}$ as $\mathsf{A}\varphi := \mathsf{Kh}(\neg\varphi,\bot)$. This is justified by the following proposition, whose proof relies on the fact that Act^* is never empty (it always contains ϵ).

Proposition 2.1 ([30]) *Let $\mathcal{S},w$ be a pointed* LTS. *Then, $\mathcal{S},w \models \mathsf{Kh}(\neg\varphi,\bot)$ iff $[\![\varphi]\!]^{\mathcal{S}} = \mathrm{D}_{\mathcal{S}}$.* ◂

Block $\mathcal{L}$:	TAUT	$\vdash \varphi$ for φ a propositional tautology	MP	From $\vdash \varphi$ and $\vdash \varphi \to \psi$ infer $\vdash \psi$
	DISTA	$\vdash \mathsf{A}(\varphi \to \psi) \to (\mathsf{A}\varphi \to \mathsf{A}\psi)$	NECA	From $\vdash \varphi$ infer $\vdash \mathsf{A}\varphi$
	TA	$\vdash \mathsf{A}\varphi \to \varphi$		
	4KhA	$\vdash \mathsf{Kh}(\psi,\varphi) \to \mathsf{AKh}(\psi,\varphi)$	5KhA	$\vdash \neg\mathsf{Kh}(\psi,\varphi) \to \mathsf{A}\neg\mathsf{Kh}(\psi,\varphi)$
Block $\mathcal{L}_{\mathrm{LTS}}$:	EMP	$\vdash \mathsf{A}(\psi \to \varphi) \to \mathsf{Kh}(\psi,\varphi)$	COMPKh	$\vdash (\mathsf{Kh}(\psi,\varphi) \wedge \mathsf{Kh}(\varphi,\chi)) \to \mathsf{Kh}(\psi,\chi)$

Table 1: Axiom system $\mathcal{L}^{\mathrm{LTS}}_{\mathsf{Kh}}$, for $\mathsf{L_{Kh}}$ w.r.t. LTSs.

The axiom system $\mathcal{L}^{\mathrm{LTS}}_{\mathsf{Kh}}$ in Tab. 1 shows that A and Kh are strongly interconnected.

Theorem 1 ([30]) *$\mathcal{L}^{\mathrm{LTS}}_{\mathsf{Kh}}$ is sound and strongly complete for $\mathsf{L_{Kh}}$ w.r.t. the class of all* LTSs. ◂

Some axioms deserve comment. If A is taken as primitive, and $\mathsf{A}\varphi$ is interpreted as φ is true at every state in an LTS, then EMP states that if $\psi \to \varphi$ is a globally true implication, then given ψ the agent has the ability to make φ true. In simpler words, global ontic information turns into knowledge. One could argue that, more realistically, there are global truths in a model that are still beyond the abilities of the agent. The case of COMPKh is similar (as it also implies a certain level of omniscience) but perhaps less controvertial. It might well be that an agent knows how to make φ true given ψ, and how to make χ true given φ, but still have not workout how to put the two together to ensure that χ given ψ. As we will see in the next section, both these axioms can be correlated with strong assumptions on the uncertainty relation between plans that an agent might have.

3 Uncertainty-based semantics

The LTS-based semantics provides a possible representation of an agent's abilities: the agent knows how to achieve φ given ψ if and only if there is a plan that, when run at any ψ-state, will always complete every partial execution, ending unerringly in states satisfying φ. One could argue that this representation involves a certain level of idealization.

Consider an agent *lacking* a certain ability. In the LTS-based semantics, this can only happen when the environment does not provide the required (sequence of) action(s). But one can think of scenarios where an adequate plan exists, and yet the agent lacks the ability for a different reason. Indeed, she might *fail to distinguish* the adequate plan from a non-adequate one, in the sense of not being able to tell that, in general, those plans produce a different outcome. Consider, for example, an agent baking a cake. She might have the ability to do the nine different mixing methods (beating, blending, creaming, cutting, folding, kneading, sifting, stirring, whipping), and she might even recognize them as different actions. However, she might not be able to perfectly distinguish one from the others in the sense of not recognizing that sometimes they produce different results. In such cases, one would say that the agent does not know how to make certain cake: sometimes she gets good outcomes (when she uses the adequate mixing method) and sometimes she does not.

Indistinguishability among *basic* actions can account for the example above (with each mixing method a basic action). Still, one can also think of situations in which a more general indistinguishability *among plans* is involved. Consider the baking agent again. It is reasonable to assume that she can tell the difference between "adding milk" and "adding flour", but perhaps she does not realize the effect that *the order* for mixing ingredients might have in the final result. Here, the issue is not that she cannot distinguish between basic actions; rather, two plans are indistinguishable because the order of their actions is being considered irrelevant. For a last possibility, the agent might not know that, while opening the oven once to check whether the baking goods are done is reasonable, this must not be done in excess. In this case, the problem is not being able to tell the difference between the effect of executing an action once and executing it multiple times. Thus, even plans of *different length* might be considered indistinguishable.

The previous examples suggest that one can devise a more general representation of an agent's abilities. This representation involves taking into account not only the plans she has available (the LTS structure), but also her skills for telling two different plans apart (a form of *indistinguishability among plans*). As we will see, the use of an indistinguishability relation among plans will also let us define a natural model for a multi-agent scenario. In this setting, agents share the same set of *affordances* (provided by the actual environment), but still have different *abilities* depending on which of these affordances they have available, and how well they can tell these affordances appart.

Definition 3.1 (Uncertainty-based LTS) Let Agt be a finite non-empty set of agents. A *multi-agent uncertainty-based* LTS (LTS$^{\mathrm{U}}$) for Prop, Act and Agt is a tuple $\mathcal{M} = \langle \mathrm{W}, \mathrm{R}, \sim, \mathrm{V}\rangle$ where $\langle \mathrm{W}, \mathrm{R}, \mathrm{V}\rangle$ is an LTS and $\sim$ assigns, to each agent $i \in \mathsf{Agt}$, an equivalence *indistinguishability* relation over a non-empty set of plans $\mathrm{P}_i \subseteq \mathsf{Act}^*$. Given an LTS$^{\mathrm{U}}$ $\mathcal{M}$ and $w \in \mathrm{D}_{\mathcal{M}}$, the pair $(\mathcal{M}, w)$ (parenthesis usually dropped) is called a *pointed* LTS$^{\mathrm{U}}$. $\dashv$

Intuitively, P_i is the set of plans that agent i has at her disposal. Similarly as in classical epistemic logic, $\sim_i \subseteq \mathrm{P}_i \times \mathrm{P}_i$ describes agent i's indistinguishability. But this time, this relation is not defined over possible states of affairs, but rather over her available plans.

Remark 3.1 *The following change in notation will simplify some definitions later on, and will make the comparison with the* LTS*-based semantics clearer. Let* $\langle \mathrm{W}, \mathrm{R}, \sim, \mathrm{V}\rangle$ *be an* $\mathrm{LTS}^{\mathrm{U}}$ *and take* $i \in \mathsf{Agt}$*; for a plan* $\sigma \in \mathrm{P}_i$*, let* $[\sigma]_i$ *be its equivalence class in* $\sim_i$ *(i.e.,* $[\sigma]_i := \{\sigma' \in \mathrm{P}_i \mid \sigma \sim_i \sigma'\}$*).*

There is a one-to-one correspondence between $\sim_i$ *and its induced set of equivalence classes* $\mathsf{S}_i := \{[\sigma]_i \mid \sigma \in \mathrm{P}_i\}$*. Hence, from now on an* $\mathrm{LTS}^{\mathrm{U}}$ *will be presented as a tuple* $\langle \mathrm{W}, \mathrm{R}, \mathsf{S}, \mathrm{V}\rangle$*, with* $\mathsf{S} = \{\mathsf{S}_i \mid i \in \mathsf{Agt}\}$*. Notice the following properties:* ***(1)*** $\pi_1 \neq \pi_2 \in \mathsf{S}_i$ *implies* $\pi_1 \cap \pi_2 = \varnothing$*,* ***(2)*** $\mathrm{P}_i = \bigcup_{\pi \in \mathsf{S}_i} \pi$ *and* ***(3)*** $\varnothing \notin \mathsf{S}_i$*.* ◄

Given her uncertainty over Act^*, the abilities of an agent i depend not on what a single plan can achieve, but rather on what a set of them can guarantee.

Definition 3.2 For $\pi \subseteq \mathsf{Act}^*$ and $U \cup \{u\} \subseteq \mathrm{W}$ define $\mathrm{R}_\pi := \bigcup_{\sigma\in\pi} \mathrm{R}_\sigma$, $\mathrm{R}_\pi(u) := \bigcup_{\sigma\in\pi} \mathrm{R}_\sigma(u)$, and $\mathrm{R}_\pi(U) := \bigcup_{u\in U} \mathrm{R}_\pi(u)$. ⊣

We can now define strong executability for sets of plans.

Definition 3.3 (Strong executability) A set of plans $\pi \subseteq \mathsf{Act}^*$ is *strongly executable* at $u \in \mathrm{W}$ if and only if *every* plan $\sigma \in \pi$ is *strongly executable* at u. Hence, $\mathrm{SE}(\pi) = \bigcap_{\sigma\in\pi} \mathrm{SE}(\sigma)$ is the set of the states in W where π is strongly executable. ⊣

Definition 3.4 (Kh_i over $\mathrm{LTS}^{\mathrm{U}}$s) The satisfiability relation $\models$ between a pointed $\mathrm{LTS}^{\mathrm{U}}$ $\mathcal{M}, w$ (with $\mathcal{M} = \langle \mathrm{W}, \mathrm{R}, \mathsf{S}, \mathrm{V}\rangle$ an $\mathrm{LTS}^{\mathrm{U}}$ over Act, Prop and Agt) and formulas in the multi-agent version of L_{Kh} (denoted by $\mathsf{L}_{\mathsf{Kh}_i}$, and obtained by replacing Kh with Kh_i, $i \in \mathsf{Agt}$) is defined inductively. The atomic and Boolean cases are as before. For *knowing how* formulas,

$$\mathcal{M}, w \models \mathsf{Kh}_i(\psi, \varphi) \quad \mathit{iff}_{def} \quad \exists \pi \in \mathsf{S}_i \text{ such that } \textbf{(Kh-}\textit{1}\textbf{)}\ [\![\psi]\!]^{\mathcal{M}} \subseteq \mathrm{SE}(\pi) \text{ and } \textbf{(Kh-}\textit{2}\textbf{)}\ \mathrm{R}_\pi([\![\psi]\!]^{\mathcal{M}}) \subseteq [\![\varphi]\!]^{\mathcal{M}},$$

with $[\![\varphi]\!]^{\mathcal{M}} := \{w \in \mathrm{W} \mid \mathcal{M}, w \models \varphi\}$. ⊣

It is worth comparing Def. 2.6 and Def. 3.4. As before, $\mathsf{Kh}_i(\psi, \varphi)$ acts *globally*. Moreover, we now require *for agent* i to have a *set of plans* satisfying strong executability in every ψ-state (condition ***(Kh-1)***). Still, the set of plans should work as the single plan did before: when executed at ψ-states, it should end unerringly in states satisfying φ (condition ***(Kh-2)***).

The rest of the section is devoted to explore the properties of the logic with our new semantics. Moreover, we compare it to the well-known framework from [30, 31, 32].

3.1 Axiomatization

We start by establishing that the universal modality is again definable within $\mathsf{L}_{\mathsf{Kh}_i}$ over $\mathrm{LTS}^{\mathrm{U}}$ (it is crucial that $\mathsf{S}_i \neq \varnothing$ and $\varnothing \notin \mathsf{S}_i$, see Remark 3.1).

Proposition 3.1 *Given* $\mathcal{M}, w$ *a pointed* $\mathrm{LTS}^{\mathrm{U}}$*, then* $(\exists i \in \mathsf{Agt}, \mathcal{M}, w \models \mathsf{Kh}_i(\neg\varphi, \bot))$ *iff* $[\![\varphi]\!]^{\mathcal{M}} = \mathrm{D}_{\mathcal{M}}$*.* ◄

Hence, by taking $\mathsf{A}\varphi := \bigvee_{i\in\mathsf{Agt}} \mathsf{Kh}_i(\neg\varphi, \bot)$ (recall that Agt is non-empty and finite) and $\mathsf{E}\varphi := \neg\mathsf{A}\neg\varphi$, it turns out that formulas in $\mathcal{L}$ (first part of Tab. 1) are still valid, generalizing Kh to Kh_i. As discussed in the next section, some valid formulas in $\mathcal{L}_{\mathrm{LTS}}$ can be falsified over $\mathrm{LTS}^{\mathrm{U}}$s. But the weaker theorems of $\mathcal{L}^{\mathrm{LTS}}_{\mathsf{Kh}}$ shown in Tab. 2 (see Prop. 3.8) are still valid, and can be used to define a complete axiomatic system.

KhA can be subjected to some of the criticism that apply to EMP and COMPKh but, in our opinion, to a lesser extent. It implies certain level of idealization, as it entails that the knowing how of an agent is, in a sense, closed under global entailment. KhE on the other hand, seems plausible: if $\mathsf{Kh}_i(\psi, \varphi)$ is not trivial (given that $\mathsf{E}\psi$ holds), then $\mathsf{E}\varphi$ should be assured.

Block $\mathcal{L}_{\mathrm{LTS}^{\mathrm{U}}}$:	KhE	$\vdash (\mathsf{E}\psi \wedge \mathsf{Kh}_i(\psi,\varphi)) \to \mathsf{E}\varphi$	KhA	$\vdash (\mathsf{A}(\chi \to \psi) \wedge \mathsf{Kh}_i(\psi,\varphi) \wedge \mathsf{A}(\varphi \to \theta)) \to \mathsf{Kh}_i(\chi,\theta)$

Table 2: Axioms $\mathcal{L}_{\mathrm{LTS}^{\mathrm{U}}}$, for $\mathsf{L}_{\mathsf{Kh}_i}$ w.r.t. $\mathrm{LTS}^{\mathrm{U}}$s.

Let us define the system $\mathcal{L}^{\mathrm{LTS}^{\mathrm{U}}}_{\mathsf{Kh}_i} := \mathcal{L} + \mathcal{L}_{\mathrm{LTS}^{\mathrm{U}}}$ (Tab. 2). We will show that the system is sound and strongly complete over $\mathrm{LTS}^{\mathrm{U}}$s. The proof of soundness is rather straighforward, thus we will focus on completeness. Following [30, 32], the strategy is to build, for any $\mathcal{L}^{\mathrm{LTS}^{\mathrm{U}}}_{\mathsf{Kh}_i}$-consistent set of formulas, an $\mathrm{LTS}^{\mathrm{U}}$ satisfying them. Note:

Proposition 3.2 *The following are theorems of $\mathcal{L}^{\mathrm{LTS}^{\mathrm{U}}}_{\mathsf{Kh}_i}$:*

SCOND: $\vdash \mathsf{A}\neg\psi \to \mathsf{Kh}_i(\psi,\varphi)$; COND: $\vdash \mathsf{Kh}_i(\bot,\varphi)$. ◀

We proceed with the definition of the *canonical model*.

Definition 3.5 (Canonical model) Let $\mathbf{\Phi}$ be the set of all maximally $\mathcal{L}^{\mathrm{LTS}^{\mathrm{U}}}_{\mathsf{Kh}_i}$-consistent sets (MCS) of formulas in $\mathsf{L}_{\mathsf{Kh}_i}$. For any $\Delta \in \mathbf{\Phi}$, define

$$\Delta|_{\mathsf{Kh}_i} := \{\xi \in \Delta \mid \xi \text{ is of the form } \mathsf{Kh}_i(\psi,\varphi)\}, \qquad \Delta|_{\mathsf{Kh}} := \textstyle\bigcup_{i\in\mathsf{Agt}} \Delta|_{\mathsf{Kh}_i}.$$

Let Γ be a set in $\mathbf{\Phi}$. Define, for each agent $i \in \mathsf{Agt}$, the set of basic actions $\mathsf{Act}^{\Gamma}_i := \{\langle\psi,\varphi\rangle \mid \mathsf{Kh}_i(\psi,\varphi) \in \Gamma\}$, and $\mathsf{Act}^{\Gamma} := \bigcup_{i\in\mathsf{Agt}} \mathsf{Act}^{\Gamma}_i$. Notice that COND implies that $\mathsf{Kh}_i(\bot,\bot) \in \Gamma$ for every $i \in \mathsf{Agt}$; since there is at least one agent, this implies that Act^{Γ} is non-empty, and thus it is an adequate set of actions. It is worth noticing that the set Act^{Γ} fixes a new signature. However, since the operators of the language cannot see the names of the actions, we can define a mapping from Act^{Γ} to any particular Act, to preserve the original signature.
Then, the structure $\mathcal{M}^{\Gamma}$, defined over Act^{Γ}, Agt and Prop, is the tuple $\langle \mathrm{W}^{\Gamma}, \mathrm{R}^{\Gamma}, \{\mathrm{S}^{\Gamma}_i\}_{i\in\mathsf{Agt}}, \mathrm{V}^{\Gamma}\rangle$ where

- $\mathrm{W}^{\Gamma} := \{\Delta \in \mathbf{\Phi} \mid \Delta|_{\mathsf{Kh}} = \Gamma|_{\mathsf{Kh}}\}$,
- $\mathrm{R}^{\Gamma}_{\langle\psi,\varphi\rangle} := \bigcup_{i\in\mathsf{Agt}} \mathrm{R}^{\Gamma}_{\langle\psi,\varphi\rangle^i}$, with $\mathrm{R}^{\Gamma}_{\langle\psi,\varphi\rangle^i} := \{(\Delta_1,\Delta_2) \in \mathrm{W}^{\Gamma} \times \mathrm{W}^{\Gamma} \mid \mathsf{Kh}_i(\psi,\varphi) \in \Gamma, \psi \in \Delta_1, \varphi \in \Delta_2\}$,
- $\mathrm{S}^{\Gamma}_i := \left\{\{\langle\psi,\varphi\rangle\} \mid \langle\psi,\varphi\rangle \in \mathsf{Act}^{\Gamma}_i\right\}$,
- $V^{\Gamma}(\Delta) := \{p \in \mathsf{Prop} \mid p \in \Delta\}$. ⊣

If $\Gamma \in \mathbf{\Phi}$, then $\mathcal{M}^{\Gamma}$ is a structure of the required type.

Proposition 3.3 *The structure $\mathcal{M}^{\Gamma} = \langle \mathrm{W}^{\Gamma}, \mathrm{R}^{\Gamma}, \{\mathrm{S}^{\Gamma}_i\}_{i\in\mathsf{Agt}}, \mathrm{V}^{\Gamma}\rangle$ is an* $\mathrm{LTS}^{\mathrm{U}}$.

Proof. It is enough to show that each S^{Γ}_i defines a partition over a non-empty subset of $\wp(\mathsf{Act}^*)$. First, COND implies $\mathsf{Kh}_i(\bot,\bot) \in \Gamma$, so $\langle\bot,\bot\rangle \in \mathsf{Act}^{\Gamma}_i$ and hence $\{\langle\bot,\bot\rangle\} \in \mathrm{S}^{\Gamma}_i$; thus, $\bigcup_{\pi\in\mathrm{S}_i} \pi \neq \varnothing$. Then, S_i indeed defines a partition over $\bigcup_{\pi\in\mathrm{S}_i} \pi$: its elements are mutually disjoint (they are singletons with different elements), collective exhaustiveness is immediate and, finally, $\varnothing \notin \mathrm{S}^{\Gamma}_i$. ■

Let $\Gamma \in \mathbf{\Phi}$, the following properties of $\mathcal{M}^{\Gamma}$ are useful (proofs are similar to the ones in [32]).

Proposition 3.4 *For any $\Delta_1, \Delta_2 \in \mathrm{W}^{\Gamma}$ we have $\Delta_1|_{\mathsf{Kh}} = \Delta_2|_{\mathsf{Kh}}$.* ◀

Proposition 3.5 *Take $\Delta \in \mathrm{W}^{\Gamma}$. If Δ has a $\mathrm{R}^{\Gamma}_{\langle\psi,\varphi\rangle}$-successor, then every $\Delta' \in \mathrm{W}^{\Gamma}$ with $\varphi \in \Delta'$ can be $\mathrm{R}^{\Gamma}_{\langle\psi,\varphi\rangle}$-reached from Δ.* ◀

Proposition 3.6 *Let φ be an $\mathsf{L}_{\mathsf{Kh}_i}$-formula. If $\varphi \in \Delta$ for every $\Delta \in \mathrm{W}^{\Gamma}$, then $\mathsf{A}\varphi \in \Delta$ for every $\Delta \in \mathrm{W}^{\Gamma}$.* ◀

Proposition 3.7 *Take ψ, ψ', φ' in $\mathsf{L}_{\mathsf{Kh}_i}$. Suppose that every $\Delta \in \mathrm{W}^\Gamma$ with $\psi \in \Delta$ has a $\mathrm{R}^\Gamma_{\langle\psi',\varphi'\rangle}$-successor. Then, $\mathsf{A}(\psi \to \psi') \in \Delta$ for all $\Delta \in \mathrm{W}^\Gamma$.* ◀

With these properties at hand, we can prove the truth lemma for $\mathcal{M}^\Gamma$.

Lemma 3.1 (Truth lemma for $\mathcal{M}^\Gamma$) *Given $\Gamma \in \mathbf{\Phi}$, take $\mathcal{M}^\Gamma = \langle \mathrm{W}^\Gamma, \mathrm{R}^\Gamma, \{\mathrm{S}^\Gamma_i\}_{i\in\mathsf{Agt}}, \mathrm{V}^\Gamma\rangle$. Then, for every $\Theta \in \mathrm{W}^\Gamma$ and every $\varphi \in \mathsf{L}_{\mathsf{Kh}_i}$, $\mathcal{M}^\Gamma, \Theta \models \varphi$ if and only if $\varphi \in \Theta$.*

Proof. The proof is by induction on φ, with the atom and Boolean cases as usual. For the rest:

Case $\mathsf{Kh}_i(\psi,\varphi)$. (⇒) Suppose $\mathcal{M}^\Gamma, \Theta \models \mathsf{Kh}_i(\psi,\varphi)$, then consider two cases.

- $[\![\psi]\!]^{\mathcal{M}^\Gamma} = \varnothing$. Then, for each $\Delta \in \mathrm{W}^\Gamma$ we have $\Delta \notin [\![\psi]\!]^{\mathcal{M}^\Gamma}$, so $\psi \notin \Delta$ (by IH) and thus $\neg\psi \in \Delta$ (by maximal consistency). Hence, by Prop. 3.6, $\mathsf{A}\neg\psi \in \Delta$ for every $\Delta \in \mathrm{W}^\Gamma$. In particular, $\mathsf{A}\neg\psi \in \Theta$ and thus, by SCOND and MP, $\mathsf{Kh}_i(\psi,\varphi) \in \Theta$.
- $[\![\psi]\!]^{\mathcal{M}^\Gamma} \neq \varnothing$. By hypothesis, there is $\{\langle\psi',\varphi'\rangle\} \in \mathrm{S}^\Gamma_i$ with ***(Kh-1)*** $[\![\psi]\!]^{\mathcal{M}^\Gamma} \subseteq \mathrm{SE}(\{\langle\psi',\varphi'\rangle\})$ and ***(Kh-2)*** $\mathrm{R}^\Gamma_{\{\langle\psi',\varphi'\rangle\}}([\![\psi]\!]^{\mathcal{M}^\Gamma}) \subseteq [\![\varphi]\!]^{\mathcal{M}^\Gamma}$. In other words, there is $\langle\psi',\varphi'\rangle \in \mathsf{Act}^\Gamma_a$ such that

  (Kh-1) for all $\Delta \in \mathrm{W}^\Gamma$, if $\Delta \in [\![\psi]\!]^{\mathcal{M}^\Gamma}$ then $\Delta \in \mathrm{SE}(\{\langle\psi',\varphi'\rangle\})$, so $\Delta \in \mathrm{SE}(\langle\psi',\varphi'\rangle)$ and therefore Δ has a $\mathrm{R}^\Gamma_{\langle\psi',\varphi'\rangle}$-successor.

  (Kh-2) for all $\Delta' \in \mathrm{W}^\Gamma$, if there is $\Delta \in [\![\psi]\!]^{\mathcal{M}^\Gamma}$ such that $(\Delta,\Delta') \in \mathrm{R}^\Gamma_{\langle\psi',\varphi'\rangle}$, then $\Delta' \in [\![\varphi]\!]^{\mathcal{M}^\Gamma}$.

  This case requires three pieces.

  (1) Take any $\Delta \in \mathrm{W}^\Gamma$ with $\psi \in \Delta$. Then, by IH, $\Delta \in [\![\psi]\!]^{\mathcal{M}^\Gamma}$ and thus, by ***(Kh-1)***, Δ has a $\mathrm{R}^\Gamma_{\langle\psi',\varphi'\rangle}$-successor. Thus, every $\Delta \in \mathrm{W}^\Gamma$ with $\psi \in \Delta$ has such successor; then (Prop. 3.7), it follows that $\mathsf{A}(\psi \to \psi') \in \Delta$ for every $\Delta \in \mathrm{W}^\Gamma$. In particular, $\mathsf{A}(\psi \to \psi') \in \Theta$.

  (2) From $\langle\psi',\varphi'\rangle \in \mathsf{Act}^\Gamma_i$ it follows that $\mathsf{Kh}_i(\psi',\varphi') \in \Gamma$. But $\Theta \in \mathrm{W}^\Gamma$, so $\Theta|_{\mathsf{Kh}} = \Gamma|_{\mathsf{Kh}}$ (by definition of W^Γ). Hence, $\mathsf{Kh}_i(\psi',\varphi') \in \Theta$.

  (3) Since $[\![\psi]\!]^{\mathcal{M}^\Gamma} \neq \varnothing$, there is $\Delta \in [\![\psi]\!]^{\mathcal{M}^\Gamma}$. By ***(Kh-1)***, Δ should have at least one $\mathrm{R}^\Gamma_{\langle\psi',\varphi'\rangle}$-successor. Then, by Prop. 3.5, every $\Delta' \in \mathrm{W}^\Gamma$ satisfying $\varphi' \in \Delta'$ can be $\mathrm{R}^\Gamma_{\langle\psi',\varphi'\rangle}$-reached from Δ; in other words, every $\Delta' \in \mathrm{W}^\Gamma$ satisfying $\varphi' \in \Delta'$ is in $\mathrm{R}^\Gamma_{\langle\psi',\varphi'\rangle}(\Delta)$. But $\Delta \in [\![\psi]\!]^{\mathcal{M}^\Gamma}$, so every $\Delta' \in \mathrm{W}^\Gamma$ satisfying $\varphi' \in \Delta'$ is in $\mathrm{R}^\Gamma_{\langle\psi',\varphi'\rangle}([\![\psi]\!]^{\mathcal{M}^\Gamma})$. Then, by ***(Kh-2)***, every $\Delta' \in \mathrm{W}^\Gamma$ satisfying $\varphi' \in \Delta'$ is in $[\![\varphi]\!]^{\mathcal{M}^\Gamma}$. By IH on the latter part, every $\Delta' \in \mathrm{W}^\Gamma$ satisfying $\varphi' \in \Delta'$ is such that $\varphi \in \Delta'$. Thus, $\varphi' \to \varphi \in \Delta'$ for every $\Delta' \in \mathrm{W}^\Gamma$, and hence (Prop. 3.6) $\mathsf{A}(\varphi' \to \varphi) \in \Delta'$ for every $\Delta' \in \mathrm{W}^\Gamma$. In particular, $\mathsf{A}(\varphi' \to \varphi) \in \Theta$.

  Thus, $\{\mathsf{A}(\psi \to \psi'), \mathsf{Kh}_i(\psi',\varphi'), \mathsf{A}(\varphi' \to \varphi)\} \subset \Theta$. Therefore, by KhA and MP, $\mathsf{Kh}_i(\psi,\varphi) \in \Theta$.

(⇐) Suppose $\mathsf{Kh}_i(\psi,\varphi) \in \Theta$. Thus (Prop. 3.4), $\mathsf{Kh}_i(\psi,\varphi) \in \Gamma$, so $\langle\psi,\varphi\rangle \in \mathsf{Act}^\Gamma_i$ and therefore $\{\langle\psi,\varphi\rangle\} \in \mathrm{S}^\Gamma_i$. The rest of the proof is split in two cases.

- Suppose there is no $\Delta_\psi \in \mathrm{W}^\Gamma$ with $\psi \in \Delta$. Then, by IH, there is no $\Delta_\psi \in \mathrm{W}^\Gamma$ with $\Delta_\psi \in [\![\psi]\!]^{\mathcal{M}^\Gamma}$, that is, $[\![\neg\psi]\!]^{\mathcal{M}^\Gamma} = \mathrm{D}_{\mathrm{W}^\Gamma}$. Since $\mathcal{M}^\Gamma$ is in $\mathrm{LTS}^{\mathrm{U}}$ (Prop. 3.3), the latter yields $(\mathcal{M}^\Gamma, \Delta) \models \mathsf{Kh}_i(\psi,\chi)$ for any $i \in \mathsf{Agt}$, $\chi \in \mathsf{L}_{\mathsf{Kh}_i}$ and $\Delta \in \mathrm{W}^\Gamma$ (cf. Prop. 3.1); hence, $(\mathcal{M}^\Gamma, \Theta) \models \mathsf{Kh}_i(\psi,\varphi)$.
- Suppose there is $\Delta_\psi \in \mathrm{W}^\Gamma$ with $\psi \in \Delta_\psi$. It will be shown that the strategy $\{\langle\psi,\varphi\rangle\} \in \mathrm{S}^\Gamma_i$ satisfies the requirements.

  (Kh-1) Take any $\Delta \in [\![\psi]\!]^{\mathcal{M}^\Gamma}$. By IH, $\psi \in \Delta$. Moreover, from $\mathsf{Kh}_i(\psi,\varphi) \in \Theta$ and Prop. 3.4 it follows that $\mathsf{Kh}_i(\psi,\varphi) \in \Delta$. Then, from $\mathrm{R}^\Gamma_{\langle\psi,\varphi\rangle^i}$'s definition, every $\Delta' \in \mathrm{W}^\Gamma$ with $\varphi \in \Delta'$ is such that $(\Delta,\Delta') \in \mathrm{R}^\Gamma_{\langle\psi,\varphi\rangle^i}$, and therefore such that $(\Delta,\Delta') \in \mathrm{R}^\Gamma_{\langle\psi,\varphi\rangle}$. Now note how, since there is $\Delta_\psi \in W^\Gamma$ with $\psi \in \Delta_\psi$, there should be $\Delta_\varphi \in W^\Gamma$ with $\varphi \in \Delta_\varphi$ (the proof uses KhE and TA). This implies that $(\Delta, \Delta_\varphi) \in R^\Gamma_{\langle\psi,\varphi\rangle}$ and thus, since $\langle\psi,\varphi\rangle$ is a basic action, $\Delta \in SE(\langle\psi,\varphi\rangle)$ so $\Delta \in SE(\{\langle\psi,\varphi\rangle\})$. Since Δ is an arbitrary state in $[\![\psi]\!]^{\mathcal{M}^\Gamma}$, the required $[\![\psi]\!]^{\mathcal{M}^\Gamma} \subseteq SE(\{\langle\psi,\varphi\rangle\})$ follows.

(Kh-2) Take any $\Delta' \in R^\Gamma_{\{\langle\psi,\varphi\rangle\}}([\![\psi]\!]^{\mathcal{M}^\Gamma})$. Then, there is $\Delta \in [\![\psi]\!]^{\mathcal{M}^\Gamma}$ such that $(\Delta, \Delta') \in R^\Gamma_{\langle\psi,\varphi\rangle}$. By definition of R^Γ, it follows that $\varphi \in \Delta'$ so, by IH, $\Delta' \in [\![\varphi]\!]^{\mathcal{M}^\Gamma}$. Since Δ' is an arbitrary state in $R^\Gamma_{\{\langle\psi,\varphi\rangle\}}([\![\psi]\!]^{\mathcal{M}^\Gamma})$, the required $R^\Gamma_{\{\langle\psi,\varphi\rangle\}}([\![\psi]\!]^{\mathcal{M}^\Gamma}) \subseteq [\![\varphi]\!]^{\mathcal{M}^\Gamma}$ follows. ■

Theorem 2 *The axiom system* $\mathcal{L}^{\mathrm{LTS^U}}_{\mathsf{Kh}_i} := \mathcal{L} + \mathcal{L}_{\mathrm{LTS^U}}$ *(Tab. 2) is sound and strongly complete for* $\mathsf{L}_{\mathsf{Kh}_i}$ *w.r.t. the class of all* $\mathrm{LTS^U}$*s.*

Proof. Take any $\mathcal{L}^{\mathrm{LTS^U}}_{\mathsf{Kh}_i}$-consistent set of formulas $\Gamma' \subseteq \mathsf{L}_{\mathsf{Kh}_i}$. Since $\mathsf{L}_{\mathsf{Kh}_i}$ is enumerable, Γ' can be extended into a maximally $\mathcal{L}^{\mathrm{LTS^U}}_{\mathsf{Kh}_i}$-consistent set $\Gamma \supseteq \Gamma'$ by a standard Lindenbaum's construction (see, e.g., [6, Chapter 4]). By Lemma 3.1, Γ' is satisfiable in $\mathcal{M}^\Gamma$ at Γ. The fact that $\mathcal{M}^\Gamma$ is in $\mathrm{LTS^U}$ (Prop. 3.3) completes the proof. ■

3.2 Comparing LTS semantics and LTS$^{\mathrm{U}}$ semantics

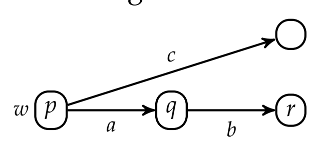
The provided axiom system can be used to compare the notion of *knowing how* under LTSs with that under $\mathrm{LTS^U}$s. Here is a first observation.

Proposition 3.8 *Axioms* KhE *and* KhA *are* $\mathcal{L}^{\mathrm{LTS}}_{\mathsf{Kh}}$*-derivable (thus,* $\mathcal{L}^{\mathrm{LTS^U}}_{\mathsf{Kh}_i}$ *is a subsystem of* $\mathcal{L}^{\mathrm{LTS}}_{\mathsf{Kh}}$*).* ◂

Hence, the *knowing how* operator under LTS is at least as strong as its $\mathrm{LTS^U}$-based counterpart: every formula valid under $\mathrm{LTS^U}$s is also valid under LTSs. The following fact shows that the converse is not the case.

Proposition 3.9 *Within* $\mathrm{LTS^U}$*, axioms* EMP *and* COMPKh *are not valid.*

Proof. Consider the $\mathrm{LTS^U}$ $\mathcal{M}$ shown below, with the collection of sets of available plans for agent i (i.e., the set S_i) depicted on the right. Recall that Kh_i acts globally.



$$S_i = \left\{ \begin{array}{c} \{a\},\ \{b\} \\ \{ab, c\} \end{array} \right\}$$

With respect to EMP, notice that $\mathsf{A}(p \to p)$ holds; yet, $\mathsf{Kh}_i(p,p)$ fails since there is no $\pi \in S_i$ leading from p-states to p-states. More generally, EMP is valid over LTSs because the empty plan ϵ, strongly executable everywhere, is always available. However, in a $\mathrm{LTS^U}$, the plan ϵ might not be available to the agent (i.e., $\epsilon \notin P_i$); and even if ϵ is avaibable, it might be indistinguishable from other plans with different behaviour.

With respect to COMPKh, notice that $\mathsf{Kh}_i(p,q)$ and $\mathsf{Kh}_i(q,r)$ hold, witness $\{a\}$ and $\{b\}$, resp. However, there is no $\pi \in S_i$ containing only plans that, when starting on p-states, lead only to r-states. This is due to the fact that, although ab acts as needed, it cannot be distinguished from c, which behaves differently. Thus, $\mathsf{Kh}_i(p,r)$ fails. More generally, COMPKh is valid over LTS because the sequential composition of the plans that make true the conjuncts in the antedecent is a witness that makes true the consequent. However, in an $\mathrm{LTS^U}$, this composition might be unavailable or else indistinguishable from other plans. ■

From these two observations it follows that Kh under LTS^{U}s is strictly weaker than Kh under LTSs: adding uncertainty about the effect of actions does change the logic. However, the LTS^{U} framework is general enough to capture the LTS semantics. To establish the connection, let us work in a single-agent setting (i.e., with a single modality Kh and no subindexes for P_i and S_i).

Given the discussion in Prop. 3.9, it should be clear that there is an obvious class of LTS^{U}s in which EMP and COMPKh are valid. This is the class of LTS^{U}s in which the agent has every plan available and can distinguish between any two of them (i.e., $\mathrm{S} = \{\{\sigma\} \mid \sigma \in \mathsf{Act}^*\}$). This is because, in such models, ϵ is available and distinguishable from other plans (for EMP) and from $\{\sigma_1\} \in \mathrm{S}$ and $\{\sigma_2\} \in \mathrm{S}$ it follows that $\{\sigma_1\sigma_2\} \in \mathrm{S}$ (for COMPKh). Clearly, other, more general, classes can be defined, but the one introduced here serves as an example.

Proposition 3.10 *Let $\mathcal{S} = \langle \mathrm{W}, \mathrm{R}, \mathrm{V} \rangle$ be an LTS over* Act*, define $\mathcal{M}_{\mathcal{S}} = \langle \mathrm{W}, \mathrm{R}, \mathrm{S}, \mathrm{V} \rangle$, where $\mathrm{S} = \{\{\sigma\} \mid \sigma \in \mathsf{Act}^*\}$. Let $C := \{\mathcal{M}_{\mathcal{S}} \mid \mathcal{S}$ is an LTS$\}$. Given $\mathcal{M} = \langle \mathrm{W}, \mathrm{R}, \mathrm{S}, \mathrm{V} \rangle$ an LTS^{U} in C, define $\mathcal{S}_{\mathcal{M}} = \langle \mathrm{W}, \mathrm{R}, \mathrm{V} \rangle$. Then, for every $\varphi \in \mathsf{L}_{\mathsf{Kh}}$, $[\![\varphi]\!]^{\mathcal{S}} = [\![\varphi]\!]^{\mathcal{M}_{\mathcal{S}}}$ and $[\![\varphi]\!]^{\mathcal{M}} = [\![\varphi]\!]^{\mathcal{S}_{\mathcal{M}}}$.* ◀

Since we have a class of LTS^{U}s in correspondence with the class of all LTSs, we get a direct completeness result:

Theorem 3 *The axiom system $\mathcal{L}^{\text{LTS}}_{\mathsf{Kh}}$ (Tab. 1) is sound and strongly complete for L_{Kh} w.r.t. the class C.*◀

3.3 Complexity

Here we investigate the computational complexity of the satisfiability problem of $\mathsf{L}_{\mathsf{Kh}_i}$ under the LTS^{U}-based semantics. We will establish membership in NP by showing a polynomial size model property.

Given a formula, we will show that it is possible to select just a piece of the canonical model which is relevant for its evaluation. The selected model will preserve satisfiability, and moreover, its size will be polymonial w.r.t. the size of the input formula.

Definition 3.6 (Selection function) Let $\mathcal{M}^\Gamma = \langle \mathrm{W}^\Gamma, \mathrm{R}^\Gamma, \{\mathrm{S}^\Gamma_i\}_{i \in \mathsf{Agt}}, \mathrm{V}^\Gamma \rangle$ be a canonical model for an MCS Γ (see Def. 3.5); take an MCS $w \in \mathrm{W}^\Gamma$ and a formula $\varphi \in \mathsf{L}_{\mathsf{Kh}_i}$. Define $\mathsf{Act}_\varphi := \{\langle \theta_1, \theta_2 \rangle \in \mathsf{Act}^\Gamma \mid \mathsf{Kh}_i(\theta_1, \theta_2)$ is a subformula of $\varphi\}$. A *canonical selection function* sel^φ_w is a function that takes $\mathcal{M}^\Gamma$, w and φ as input, returns a set $\mathrm{W}' \subseteq \mathrm{W}^\Gamma$, and is s.t.:

(1) $\mathsf{sel}^\varphi_w(p) = \{w\}$; $\mathsf{sel}^\varphi_w(\neg\varphi_1) = \mathsf{sel}^\varphi_w(\varphi_1)$; $\mathsf{sel}^\varphi_w(\varphi_1 \vee \varphi_2) = \mathsf{sel}^\varphi_w(\varphi_1) \cup \mathsf{sel}^\varphi_w(\varphi_2)$;

(2) If $[\![\mathsf{Kh}_i(\varphi_1, \varphi_2)]\!]^{\mathcal{M}^\Gamma} \neq \varnothing$ and $[\![\varphi_1]\!]^{\mathcal{M}^\Gamma} = \varnothing$: $\mathsf{sel}^\varphi_w(\mathsf{Kh}_i(\varphi_1, \varphi_2)) = \{w\}$;

(3) If $[\![\mathsf{Kh}_i(\varphi_1, \varphi_2)]\!]^{\mathcal{M}^\Gamma} \neq \varnothing$ and $[\![\varphi_1]\!]^{\mathcal{M}^\Gamma} \neq \varnothing$:
$\mathsf{sel}^\varphi_w(\mathsf{Kh}_i(\varphi_1, \varphi_2)) = \{w_1, w_2\} \cup \mathsf{sel}^\varphi_{w_1}(\varphi_1) \cup \mathsf{sel}^\varphi_{w_2}(\varphi_2)$, where w_1, w_2 are s.t. $(w_1, w_2) \in \mathrm{R}^\Gamma_{\langle \varphi_1, \varphi_2 \rangle}$;

(4) If $[\![\mathsf{Kh}_i(\varphi_1, \varphi_2)]\!]^{\mathcal{M}^\Gamma} = \varnothing$ (note that $[\![\varphi_1]\!]^{\mathcal{M}^\Gamma} \neq \varnothing$):
For all set of plans π, either $[\![\varphi_1]\!]^{\mathcal{M}^\Gamma} \not\subseteq \mathrm{SE}(\pi)$ or $\mathrm{R}^\Gamma_\pi([\![\varphi_1]\!]^{\mathcal{M}^\Gamma}) \not\subseteq [\![\varphi_2]\!]^{\mathcal{M}^\Gamma}$. For each $a \in \mathsf{Act}_\varphi$:

- ***(a)*** if $[\![\varphi_1]\!]^{\mathcal{M}^\Gamma} \not\subseteq \mathrm{SE}(\{a\})$: we add $\{w_1\} \cup \mathsf{sel}^\varphi_{w_1}(\varphi_1)$ to $\mathsf{sel}^\varphi_w(\mathsf{Kh}_i(\varphi_1, \varphi_2))$, where $w_1 \in [\![\varphi_1]\!]^{\mathcal{M}^\Gamma}$ and $w_1 \notin \mathrm{SE}(\{a\})$;
- ***(b)*** if $\mathrm{R}^\Gamma_\pi([\![\varphi_1]\!]^{\mathcal{M}^\Gamma}) \not\subseteq [\![\varphi_2]\!]^{\mathcal{M}^\Gamma}$ we add $\{w_1, w_2\} \cup \mathsf{sel}^\varphi_{w_1}(\varphi_1) \cup \mathsf{sel}^\varphi_{w_2}(\varphi_2)$ to $\mathsf{sel}^\varphi_w(\mathsf{Kh}_i(\varphi_1, \varphi_2))$, where $w_1 \in [\![\varphi_1]\!]^{\mathcal{M}^\Gamma}$, $w_2 \in \mathrm{R}^\Gamma_a(w_1)$ and $w_2 \notin [\![\varphi_2]\!]^{\mathcal{M}^\Gamma}$. ⊣

We can now select a small model which preserves the satisfiability of a given formula.

Definition 3.7 (Selected model) Let $\mathcal{M}^\Gamma$ be the canonical model for an MCS Γ, w a state in $\mathcal{M}^\Gamma$, and φ an $\mathsf{L}_{\mathsf{Kh}_i}$-formula. Let sel^φ_w be a selection function, we define the *model selected by* sel^φ_w as $\mathcal{M}^\varphi_w = \langle \mathrm{W}^\varphi_w, \mathrm{R}^\varphi_w, \{(\mathrm{S}^\varphi_w)_i\}_{i\in\mathsf{Agt}}, \mathrm{V}^\varphi_w\rangle$, where

- $\mathrm{W}^\varphi_w := \mathsf{sel}^\varphi_w(\varphi)$;
- $(\mathrm{R}^\varphi_w)_{\langle\theta_1,\theta_2\rangle} := \mathrm{R}^\Gamma_{\langle\theta_1,\theta_2\rangle} \cap (\mathrm{W}^\varphi_w)^2$ for each $\langle\theta_1,\theta_2\rangle \in \mathsf{Act}_\varphi$;
- $(\mathrm{S}^\varphi_w)_i := \{\{a\} \mid a \in \mathsf{Act}_\varphi\} \cup \{\{\langle\bot,\top\rangle\}\}$, for $i \in \mathsf{Agt}$ (and $(\mathrm{R}^\varphi_w)_{\langle\bot,\top\rangle} := \varnothing$);
- V^φ_w is the restriction of V^Γ to W^φ_w. ⊣

Note that, although Act_φ can be an empty set, each collection of sets of plans $(\mathrm{S}^\varphi_w)_i$ is not. Therefore, $\mathcal{M}^\varphi_w$ is an LTS^U.

Proposition 3.11 *Let $\mathcal{M}^\Gamma$ be a canonical model, w a state in $\mathcal{M}^\Gamma$ and φ an $\mathsf{L}_{\mathsf{Kh}_i}$-formula. Let $\mathcal{M}^\varphi_w$ be the selected model by a selection function sel^φ_w. $\mathcal{M}^\Gamma, w \models \varphi$ implies that for all ψ subformula of φ, and for all $v \in \mathrm{W}^\varphi_w$ we have that $\mathcal{M}^\Gamma, v \models \psi$ iff $\mathcal{M}^\varphi_w, v \models \psi$. Moreover, $\mathcal{M}^\varphi_w$ is polynomial on the size of φ.* ◀

Proof. The proof proceeds by induction in the size of the formula. Boolean cases are simple, so we will proceed with the case in which $\psi = \mathsf{Kh}_i(\psi_1,\psi_2)$.

Suppose that $\mathcal{M}^\Gamma, v \models \mathsf{Kh}_i(\psi_1,\psi_2)$. Then, we have two cases:

- $[\![\psi_1]\!]^{\mathcal{M}^\Gamma} \neq \varnothing$: by $\mathcal{M}^\Gamma, v \models \mathsf{Kh}_i(\psi_1,\psi_2)$, there exists a $\pi \in \mathrm{S}^\Gamma_i$ s.t. $[\![\psi_1]\!]^{\mathcal{M}^\Gamma} \subseteq \mathrm{SE}^{\mathcal{M}^\Gamma}(\pi)$ and $\mathrm{R}^\Gamma_\pi([\![\psi_1]\!]^{\mathcal{M}^\Gamma}) \subseteq [\![\psi_2]\!]^{\mathcal{M}^\Gamma}$. By Truth Lemma, $\mathsf{Kh}_i(\psi_1,\psi_2) \in v$, then $\mathsf{Kh}_i(\psi_1,\psi_2) \in \Gamma$ and $\langle\psi_1,\psi_2\rangle \in \mathsf{Act}_\Gamma$. By the definition of $\mathrm{R}^\Gamma_{\langle\psi_1,\psi_2\rangle}$, we have that for all $w \in [\![\psi_1]\!]^{\mathcal{M}^\Gamma}$, it holds that $\mathrm{R}^\Gamma_{\langle\psi_1,\psi_2\rangle}(w) \neq \varnothing$ and $\mathrm{R}^\Gamma_{\langle\psi_1,\psi_2\rangle}(w) \subseteq [\![\psi_2]\!]^{\mathcal{M}^\Gamma}$. Thus, $[\![\psi_1]\!]^{\mathcal{M}^\Gamma} \subseteq \mathrm{SE}^{\mathcal{M}^\Gamma}(\{\langle\psi_1,\psi_2\rangle\})$ and $\mathrm{R}^\Gamma_{\langle\psi_1,\psi_2\rangle}([\![\psi_1]\!]^{\mathcal{M}^\Gamma}) \subseteq [\![\psi_2]\!]^{\mathcal{M}^\Gamma}$. Since $[\![\psi_1]\!]^{\mathcal{M}^\Gamma} \neq \varnothing$, there exist $w_1, w_2 \in \mathrm{W}^\Gamma$ s.t. $(w_1,w_2) \in \mathrm{R}^\Gamma_{\langle\psi_1,\psi_2\rangle}$.

  Notice that by definition of $\mathcal{M}^\varphi_w$, we have that $\{\langle\psi_1,\psi_2\rangle\} \in (\mathrm{S}^\varphi_w)_i$ and that $(\mathrm{R}^\varphi_w)_{\langle\psi_1,\psi_2\rangle}$ is defined. Also, by the definition of sel^φ_w, Item (3), there exist $w'_1, w'_2 \in \mathrm{W}^\varphi_w$ s.t. $(w'_1,w'_2) \in (\mathrm{R}^\varphi_w)_{\langle\psi_1,\psi_2\rangle}$. Let $v_1 \in [\![\psi_1]\!]^{\mathcal{M}^\varphi_w} \subseteq [\![\psi_1]\!]^{\mathcal{M}^\Gamma}$ (the inclusion holds by IH). Then, we have $v_1 \in \mathrm{SE}^{\mathcal{M}^\Gamma}(\{\langle\psi_1,\psi_2\rangle\})$ and $\mathrm{R}^\Gamma_{\langle\psi_1,\psi_2\rangle}(v_1) \subseteq [\![\psi_2]\!]^{\mathcal{M}^\Gamma}$. Since for all $v_2 \in \mathrm{R}^\Gamma_{\langle\psi_1,\psi_2\rangle}(v_1)$, we have $v_2 \in [\![\psi_2]\!]^{\mathcal{M}^\Gamma}$, (in particular $v_2 = w'_2$), then $w'_2 \in (\mathrm{R}^\varphi_w)_{\langle\psi_1,\psi_2\rangle}(v_1)$. Thus, $v_1 \in \mathrm{SE}^{\mathcal{M}^\varphi_w}(\{\langle\psi_1,\psi_2\rangle\})$.

  Aiming for a contradiction, suppose now that $(\mathrm{R}^\varphi_w)_{\langle\psi_1,\psi_2\rangle}(v_1) = \mathrm{R}^\Gamma_{\langle\psi_1,\psi_2\rangle}(v_1) \cap \mathrm{W}^\varphi_w \nsubseteq [\![\psi_2]\!]^{\mathcal{M}^\varphi_w}$; and let $v_2 \in (\mathrm{R}^\varphi_w)_{\langle\psi_1,\psi_2\rangle}(v_1)$ s.t. $v_2 \notin [\![\psi_2]\!]^{\mathcal{M}^\varphi_w}$. Then we have that $(\mathrm{R}^\varphi_w)_{\langle\psi_1,\psi_2\rangle}(v_1) \subseteq \mathrm{R}^\Gamma_{\langle\psi_1,\psi_2\rangle}(v_1)$, but also by IH $v_2 \notin [\![\psi_2]\!]^{\mathcal{M}^\Gamma}$. Thus, $\mathcal{M}^\Gamma, v \not\models \mathsf{Kh}_i(\psi_1,\psi_2)$, which is a contradiction. Then, it must be the case that $(\mathrm{R}^\varphi_w)_{\{\langle\psi_1,\psi_2\rangle\}}(v_1) \subseteq [\![\psi_2]\!]^{\mathcal{M}^\varphi_w}$. Since we showed that $[\![\psi_1]\!]^{\mathcal{M}^\varphi_w} \subseteq \mathrm{SE}^{\mathcal{M}^\varphi_w}(\{\langle\psi_1,\psi_2\rangle\})$ and $(\mathrm{R}^\varphi_w)_{\{\langle\psi_1,\psi_2\rangle\}}([\![\psi_1]\!]^{\mathcal{M}^\varphi_w}) \subseteq [\![\psi_2]\!]^{\mathcal{M}^\varphi_w}$, we can conclude $\mathcal{M}^\varphi_w, v \models \mathsf{Kh}_i(\psi_1,\psi_2)$.

- $[\![\psi_1]\!]^{\mathcal{M}^\Gamma} = \varnothing$: this case is direct.

Suppose now that $\mathcal{M}^\varphi_w, v \models \mathsf{Kh}_i(\psi_1,\psi_2)$:

- $[\![\psi_1]\!]^{\mathcal{M}^\varphi_w} \neq \varnothing$: first, notice that by IH, $[\![\psi_1]\!]^{\mathcal{M}^\Gamma} \neq \varnothing$. Also, by $\mathcal{M}^\varphi_w, v \models \mathsf{Kh}_i(\psi_1,\psi_2)$, we get $[\![\psi_1]\!]^{\mathcal{M}^\varphi_w} \subseteq \mathrm{SE}^{\mathcal{M}^\varphi_w}(\pi')$ and $(\mathrm{R}^\varphi_w)_{\pi'}([\![\psi_1]\!]^{\mathcal{M}^\varphi_w}) \subseteq [\![\psi_2]\!]^{\mathcal{M}^\varphi_w}$, for some $\pi' \in (\mathrm{S}^\varphi_w)_i$. Aiming for a contradiction, suppose $\mathcal{M}^\Gamma, v \not\models \mathsf{Kh}_i(\psi_1,\psi_2)$. This implies that for all $\pi \in \mathrm{S}^\Gamma_i$, $[\![\psi_1]\!]^{\mathcal{M}^\Gamma} \nsubseteq \mathrm{SE}^{\mathcal{M}^\Gamma}(\pi)$ or $\mathrm{R}^\Gamma_\pi([\![\psi_1]\!]^{\mathcal{M}^\Gamma}) \nsubseteq [\![\psi_2]\!]^{\mathcal{M}^\Gamma}$. Also, by definition of Act_φ we have that for all $\pi =$

$\{a\} \in (\mathrm{S}^{\varphi}_{w})_i$, with $a \in \mathsf{Act}_{\varphi}$, $[\![\psi_1]\!]^{\mathcal{M}^{\Gamma}} \not\subseteq \mathrm{SE}^{\mathcal{M}^{\Gamma}}(\pi)$ or $\mathrm{R}^{\Gamma}_{\pi}([\![\psi_1]\!]^{\mathcal{M}^{\Gamma}}) \not\subseteq [\![\psi_2]\!]^{\mathcal{M}^{\Gamma}}$; i.e., for all $a \in \mathsf{Act}_{\varphi}$ $[\![\psi_1]\!]^{\mathcal{M}^{\Gamma}} \not\subseteq \mathrm{SE}^{\mathcal{M}^{\Gamma}}(\{a\})$ or $\mathrm{R}^{\Gamma}_{\{a\}}([\![\psi_1]\!]^{\mathcal{M}^{\Gamma}}) \not\subseteq [\![\psi_2]\!]^{\mathcal{M}^{\Gamma}}$. Thus, there exists $w_1 \in [\![\psi_1]\!]^{\mathcal{M}^{\Gamma}}$ s.t. $w_1 \notin \mathrm{SE}^{\mathcal{M}^{\Gamma}}(a)$ or there exists $w_2 \in \mathrm{R}^{\Gamma}_{a}(w_1)$ s.t. $w_2 \notin [\![\psi_2]\!]^{\mathcal{M}^{\Gamma}}$. By definition of $\mathsf{sel}^{\varphi}_{w}$, Item *(4)*, we add witnesses for each $a \in \mathsf{Act}_{\varphi}$. So, let $\pi' \in (\mathrm{S}^{\varphi}_{w})_i$. If $\pi' = \{\langle \bot, \top \rangle\}$, trivially we obtain $\varnothing \neq [\![\psi_1]\!]^{\mathcal{M}^{\varphi}_{w}} \not\subseteq \mathrm{SE}^{\mathcal{M}^{\varphi}_{w}}(\pi') = \varnothing$. Then, take another $\pi' = \{a\}$ s.t. $a \in \mathsf{Act}_{\varphi}$, and $w'_1 \in [\![\psi_1]\!]^{\mathcal{M}^{\varphi}_{w}} \subseteq [\![\psi_1]\!]^{\mathcal{M}^{\Gamma}}$. If $w'_1 \notin \mathrm{SE}^{\mathcal{M}^{\Gamma}}(\{a\})$, $\mathrm{R}^{\Gamma}_{a}(w'_1) = \varnothing$ and thus $(\mathrm{R}^{\varphi}_{w})_a(w'_1) = \varnothing$ and therefore $w'_1 \notin \mathrm{SE}^{\mathcal{M}^{\varphi}_{w}}(\{a\})$. On the other hand, if there exists $w_2 \in \mathrm{R}^{\Gamma}_{a}(w'_1)$ s.t. $w_2 \notin [\![\psi_2]\!]^{\mathcal{M}^{\Gamma}}$, then by $\mathsf{sel}^{\varphi}_{w}$ and IH, there exists $w'_2 \in \mathrm{W}^{\varphi}_{w}$ s.t. $w'_2 \in \mathrm{R}^{\Gamma}_{a}(w'_1)$ and $w'_2 \notin [\![\psi_2]\!]^{\mathcal{M}^{\varphi}_{w}}$, and consequently, there exists $w'_2 \in (\mathrm{R}^{\varphi}_{w})_a(w'_1)$ s.t. $w'_2 \notin [\![\psi_2]\!]^{\mathcal{M}^{\varphi}_{w}}$. In any case, it leads to $\mathcal{M}^{\varphi}_{w}, v \not\models \mathsf{Kh}_i(\psi_1, \psi_2)$, a contradiction. Therefore, $\mathcal{M}^{\Gamma}, v \models \mathsf{Kh}_i(\psi_1, \psi_2)$.

- $[\![\psi_1]\!]^{\mathcal{M}^{\varphi}_{w}} = \varnothing$: similar to the previous case.

Thus, we proved the case $\mathcal{M}^{\Gamma}, v \models \mathsf{Kh}_i(\psi_1, \psi_2)$ iff $\mathcal{M}^{\varphi}_{w}, v \models \mathsf{Kh}_i(\psi_1, \psi_2)$. Therefore, we get that for all ψ subformula of φ and $v \in \mathrm{W}^{\varphi}_{w}$, $\mathcal{M}^{\Gamma}, v \models \psi$ iff $\mathcal{M}^{\varphi}_{w}, v \models \psi$. Notice that the selection funcion adds worlds from $\mathcal{M}^{\Gamma}$, only for each Kh_i-formula that appears as a subformula of φ. Clearly, there is a polynomial number of such subformulas. Moreover, the number of worlds added at each time is also polynomial in the size of φ. Hence, W^{φ}_{w} is of polynomial size. Since $(\mathrm{S}^{\varphi}_{w})_i$ is also polynomial, we have that the size of $\mathcal{M}^{\varphi}_{w}$ is polynomial in the size of φ.

In order to prove that the satisfiability problem of $\mathsf{L}_{\mathsf{Kh}_i}$ is in NP, it remains to show that the model checking problem is in P.

Proposition 3.12 *The model checking problem for $\mathsf{L}_{\mathsf{Kh}_i}$ is in P.* ◄

Proof. Given a pointed $\mathrm{LTS}^{\mathrm{U}}$ $\mathcal{M}, w$ and a formula φ, we define a bottom-up labeling algorithm running in polymonial time which checks whether $\mathcal{M}, w \models \varphi$. We follow the same ideas as for the basic modal logic K (see e.g., [5]). Below we introduce the case for formulas of the shape $\mathsf{Kh}_i(\psi, \varphi)$, over an $\mathrm{LTS}^{\mathrm{U}}$ $\mathcal{M} = \langle \mathrm{W}, \mathrm{R}, \mathrm{S}, \mathrm{V} \rangle$:

Procedure ModelChecking(($\mathcal{M}, w$), $\mathsf{Kh}_i(\psi, \varphi)$)
lab($\mathsf{Kh}_i(\psi, \varphi)$) $\leftarrow \varnothing$;
for all $\pi \in \mathrm{S}_i$ **do**
  kh $\leftarrow$ *True*;
  for all $\sigma \in \pi$ **do**
    for all $v \in lab(\psi)$ **do**
      $kh \leftarrow (kh \;\&\; v \in \mathrm{SE}(\sigma) \;\&\; \mathrm{R}_{\sigma}(v) \subseteq lab(\varphi))$;
    end for
  end for
  if *kh* **then**
    lab($\mathsf{Kh}_i(\psi, \varphi)$) $\leftarrow \mathrm{W}$;
  end if
end for

As S_i and each $\pi \in \mathrm{S}_i$ are not empy, the first two **for** loops are necessarily executed. If $lab(\psi) = \varnothing$, then the formula $\mathsf{Kh}_i(\psi, \varphi)$ is trivally true. Otherwise, *kh* will remain true only if the appropriate conditions for the satisfiability of $\mathsf{Kh}_i(\psi, \varphi)$) hold. If no π succeeds, then the initialization of $lab(\mathsf{Kh}_i(\psi, \varphi))$ as $\varnothing$ will not be overwritten, as it should be. Both $v \in \mathrm{SE}(\sigma)$ and R_{σ} can be verified in polynomial time. Hence, the model checking problem is in P. ■

The intended result for satisfiability now follows.

Theorem 4 *The satisfiability problem for* $\mathsf{L}_{\mathsf{Kh}_i}$ *over* $\mathrm{LTS}^{\mathrm{U}}$*s is* NP*-complete.*

Proof. Hardness follows from NP-completeness of propositional logic (a fragment of $\mathsf{L}_{\mathsf{Kh}_i}$). By Prop. 3.11, each satisfiable formula φ has a model of polynomial size on φ. Thus, we can guess a polymonial model $\mathcal{M}, w$, and verify $\mathcal{M}, w \models \varphi$ (which can be done in polyonomial time, due to Prop. 3.12). Thus, the satisfiability problem is in the class NP. ■

4 Final Remarks

In this article, we introduce a new semantics for the *knowing how* modality from [30, 31, 32], over multiple agents. It is defined in terms of *uncertainty-based labeled transition systems (*$\mathrm{LTS}^{\mathrm{U}}$*)*. The novelty in our proposal is that $\mathrm{LTS}^{\mathrm{U}}$s are equipped with an indistinguishability relation among plans. In this way, the epistemic notion of uncertainty of an agent –which in turn defines her epistemic state– is reintroduced, bringing the notion of *knowing how* closer to the notion of *knowing that* from classical epistemic logics. We believe that the semantics based on $\mathrm{LTS}^{\mathrm{U}}$ can represent properly the situation of a shared, objective description of the affordances of a given situation, together with the different, subjective and personal abilities of a group of agents; this seems difficult to achieve using a semantics based on LTSs alone.

We show that the logic of [30, 31, 32] can be obtained by imposing particular conditions over $\mathrm{LTS}^{\mathrm{U}}$s; thus, the new semantics is more general. In particular, it provides counter-examples to EMP and COMP, which directly link Kh to properties of the universal modality.[2] Indeed, consider EMP: even though $\mathsf{A}(\psi \to \varphi)$ objectively holds in the underlying LTS of an $\mathrm{LTS}^{\mathrm{U}}$, it could be argued that an agent might not have actions or plans at her disposal to turn those facts into knowledge, resulting in $\mathsf{Kh}(\psi, \varphi)$ failing on the model. Moreover, we have introduced a sound and strongly complete axiom system for the new semantics over $\mathrm{LTS}^{\mathrm{U}}$s. Finally, we showed that the satisfiability problem for our multi-agent knowing how logic over the $\mathrm{LTS}^{\mathrm{U}}$-based semantics is NP-complete, via a selection argument (and model checking is polynomial).

Future work. There are several interesting lines of research to explore in the future. First, our framework easily accommodates other notions of executability. For instance, one could require only some of the plans in a set π to be strongly executable, weaken the condition of *strong* executability, etc. We can also explore the effects of imposing different restrictions on the construction of the indistinguishability relation between plans. It would be interesting to investigate which logics we obtain in these cases, and their relations with the LTS semantics.

Second, to our knowledge, the exact complexity of the satisfiability problem for knowing how over LTSs is open. It would be interesting to see whether an adaptation of our selection procedure works over LTSs.

Third, the $\mathrm{LTS}^{\mathrm{U}}$ semantics, in the multi-agent setting, leads to natural definitions of concepts such as *global*, *distributed* and *common knowing how*, which should be investigated in detail.

Finally, dynamic modalities capturing epistemic updates can be defined via operations that modify the indistinguishability relation among plans (as is done with other dynamic epistemic operators, see, e.g., [7]). This would allow to express different forms of communication, such as *public*, *private* and *semi-private* announcements concerning (sets of) plans.

[2] The rest of the axioms and rules in $\mathcal{L}_{\mathsf{Kh}}^{\mathrm{LTS}}$ (those shown in block $\mathcal{L}$) merely state properties of the universal modality and the fact that Kh is global.

Acknowledgments. This work is partially supported by projects ANPCyT-PICT-2017-1130, Stic-AmSud 20-STIC-03 'DyLo-MPC', Secyt-UNC, GRFT Mincyt-Cba, and by the Laboratoire International Associé SINFIN.

References

[1] S. Artemov (2008): *The logic of justification*. *The Review of Symbolic Logic* 1(04), pp. 477–513, doi:10.1017/S1755020308090060.

[2] A. Baltag (2016): *To Know is to Know the Value of a Variable*. In: *Advances in Modal Logic 11*, pp. 135–155.

[3] F. Belardinelli (2014): *Reasoning about Knowledge and Strategies: Epistemic Strategy Logic*. In: *Proceedings of SR 2014*, pp. 27–33, doi:10.4204/EPTCS.146.4.

[4] J. van Benthem (2011): *Logical Dynamics of Information and Interaction*. Cambridge University Press, doi:10.1017/CBO9780511974533.

[5] P. Blackburn & J. van Benthem (2006): *Modal Logic: A Semantic Perspective*. In: *Handbook of Modal Logic*, Elsevier, pp. 1–84, doi:10.1016/s1570-2464(07)80004-8.

[6] P. Blackburn, M. de Rijke & Y. Venema (2002): *Modal Logic*. Cambridge University Press, doi:10.1017/CBO9781107050884.

[7] H. van Ditmarsch, W. van der Hoek & B. Kooi (2007): *Dynamic Epistemic Logic*. Springer, doi:10.1007/978-1-4020-5839-4.

[8] J. van Eijck, M. Gattinger & Y. Wang (2017): *Knowing Values and Public Inspection*. In: *Logic and Its Applications - 7th Indian Conference, ICLA 2017*, pp. 77–90, doi:10.1007/978-3-662-54069-5_7.

[9] R. Fagin, J. Y. Halpern, Y. Moses & M. Y. Vardi (1995): *Reasoning about knowledge*. The MIT Press, Cambridge, Mass., doi:10.7551/mitpress/5803.001.0001.

[10] J. Fan, Y. Wang & H. van Ditmarsch (2015): *Contingency and Knowing Whether*. *The Review of Symbolic Logic* 8, pp. 75–107, doi:10.1017/S1755020314000343.

[11] J. Fantl (2017): *Knowledge How*. In E. Zalta, editor: *The Stanford Encyclopedia of Philosophy*, fall 2017 edition, Metaphysics Research Lab, Stanford University.

[12] R. Fervari, A. Herzig, Y. Li & Y. Wang (2017): *Strategically knowing how*. In: *Proceedings of the Twenty-Sixth International Joint Conference on Artificial Intelligence, IJCAI 2017*, pp. 1031–1038, doi:10.24963/ijcai.2017/143.

[13] V. Goranko & S. Passy (1992): *Using the Universal Modality: Gains and Questions*. *Journal of Logic and Computation* 2(1), pp. 5–30, doi:10.1093/logcom/2.1.5.

[14] T. Gu & Y. Wang (2016): *"Knowing value" logic as a normal modal logic*. In: *Advances in Modal Logic 11*, pp. 362–381.

[15] D. Harel, D. Kozen & J. Tiuryn (2000): *Dynamic Logic*. The MIT Press, doi:10.7551/mitpress/2516.001.0001.

[16] S. Hart, A. Heifetz & D. Samet (1996): *Knowing Whether, Knowing That, and The Cardinality of State Spaces*. *Journal of Economic Theory* 70(1), pp. 249–256, doi:10.1006/jeth.1996.0084.

[17] A. Herzig (2015): *Logics of knowledge and action: critical analysis and challenges*. *Autonomous Agents and Multi-Agent Systems* 29(5), pp. 719–753, doi:10.1007/s10458-014-9267-z.

[18] A. Herzig & N. Troquard (2006): *Knowing how to play: uniform choices in logics of agency*. In: *Proceedings of AAMAS 06*, pp. 209–216, doi:10.1145/1160633.1160666.

[19] J. Hintikka (1962): *Knowledge and Belief*. Cornell University Press, Ithaca N.Y.

[20] W. van der Hoek, B. van Linder & J-J. Ch. Meyer (2000): *On Agents That Have the Ability to Choose*. *Studia Logica* 66(1), pp. 79–119, doi:10.1023/A:1026796912842.

[21] W. van der Hoek & A. Lomuscio (2003): *Ignore at your peril – towards a logic for ignorance*. In: *Proceedings of AAMAS 03*, pp. 1148–1149, doi:10.1145/860575.860839.

[22] W. Jamroga & T. Ågotnes (2007): *Constructive knowledge: what agents can achieve under imperfect information*. *Journal of Applied Non-Classical Logics* 17(4), pp. 423–475, doi:10.3166/jancl.17.423-475.

[23] W. Jamroga & W. van der Hoek (2004): *Agents that Know How to Play*. *Fundamenta Informaticae* 63(2-3), pp. 185–219.

[24] Y. Lespérance, H. J. Levesque, F. Lin & R. B. Scherl (2000): *Ability and Knowing How in the Situation Calculus*. *Studia Logica* 66(1), pp. 165–186, doi:10.1023/A:1026761331498.

[25] Y. Li (2017): *Stopping Means Achieving: A Weaker Logic of Knowing How*. *Studies in Logic* 9(4), pp. 34–54.

[26] Y. Li & Y. Wang (2017): *Achieving While Maintaining: A Logic of Knowing How with Intermediate Constraints*. In: *Logic and Its Applications - 7th Indian Conference, ICLA 2017*, pp. 154–167, doi:10.1007/978-3-662-54069-5_12.

[27] J. McCarthy & P. J. Hayes (1969): *Some Philosophical Problems from the Standpoint of Artificial Intelligence*. In: *Machine Intelligence*, Edinburgh University Press, pp. 463–502.

[28] R. Moore (1985): *A formal theory of knowledge and action*. In: *Formal Theories of the Commonsense World*, Ablex Publishing Corporation.

[29] X. Wang (2019): *A Logic of Knowing How with Skippable Plans*. In: *Logic, Rationality, and Interaction – 7th International Workshop, LORI 2019*, pp. 413–424, doi:10.1007/978-3-662-60292-8_30.

[30] Y. Wang (2015): *A Logic of Knowing How*. In: *Logic, Rationality, and Interaction – 5th International Workshop, LORI 2015*, pp. 392–405, doi:10.1007/978-3-662-48561-3_32.

[31] Y. Wang (2018): *Beyond knowing that: a new generation of epistemic logics*. In H. van Ditmarsch & G. Sandu, editors: *J. Hintikka on knowledge and game theoretical semantics*, Springer, pp. 499–533, doi:10.1007/978-3-319-62864-6_21.

[32] Y. Wang (2018): *A logic of goal-directed knowing how*. *Synthese* 195(10), pp. 4419–4439, doi:10.1007/s11229-016-1272-0.

[33] C. Xu, Y. Wang & T. Studer (2021): *A Logic of Knowing Why*. *Synthese* 198, pp. 1259–1285, doi:10.1007/s11229-019-02104-0.