

A Deontic Logic of Knowingly Complying

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ABSTRACT

We introduce a logic for representing the deontic notion of *knowingly complying* –associated to an agent’s consciousness of taking a normative course of action for achieving a certain goal. Our logic features an operator for describing normative courses of actions, and another operator for describing what each agent knowingly complies with. We provide a sound and complete axiom system for our logic, and study the computational complexity of its satisfiability problem. Finally, we extend our logic with an additional operator for capturing the general abilities of the agents. This operator enables us to distinguish ‘what agents can do’ and ‘what agents do according to norms’. For this extension, we also provide a sound and complete axiom system.

KEYWORDS

Deontic Logic, Knowingly Complying, Multiagent

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1 INTRODUCTION

Normative systems are ubiquitous in many disciplines –e.g., legal reasoning, computer science, knowledge representation, philosophy, etc. To be able to reason rigorously, and so logically, in and about normative systems is an imperative. One of the most prominent logical approaches to normative reasoning corresponds to Deontic Logics [4, 23, 30]. In brief, deontic logics are formalisms tailored to describing and reasoning about scenarios involving norms and related concepts [15, 16]. In this respect, they can be used, e.g., to determine if such normative scenarios are free of contradictions or, so-called, paradoxes.

Typically, deontic logics propose operators to speak about the obligations, permissions, and prohibitions, of some actors –generally called *agents*– involved in a certain scenario. It has been argued, e.g., in [30, 31], that these operators should range over the *actions* executed by the agents, rather than over propositions or states of affairs. Deontic logics with this characteristic are commonly known as *ought-to-do* deontic logics [1]. To a large extent, ought-to-do deontic logics focus their attention on the normative status of actions

at some specific moment –i.e., in a particular static situation. This view ignores for instance, the circumstances that lead an agent to reach a certain state, and overlooks the *course of action* that an agent may take to achieve a goal. To deal with these situations, ought-to-do deontic logics incorporate ideas coming from the world of *strategic* reasoning; see, e.g., [13, 22]. In a strategic ought-to-do deontic logic, the description of a normative situation considers not only the status of a given set of norms, but also provides a view on the behaviour of the agents with respect to their background.

In ought-to-do deontic logics, strategic reasoning is often achieved by incorporating ways of describing sequences of actions (or plans) over a temporal dimension. This temporal dimension is then used to model and study (a part of) a particular normative system. Many existing frameworks of this kind extend STIT logic [5] with temporal operators. This approach can be found, e.g., in [10, 11]. Therein, temporal operators for ‘historical necessity’, together with a standard epistemic operator of ‘knowing that’, are incorporated into a STIT logic. The resulting logical system is shown to accommodate for the notion of *knowingly doing* –whose goal is to characterize different modes of responsibility for an agent who breaks a norm (a concept known as ‘guilty mind’ or ‘mens rea’). The work in [12] takes a similar approach but incorporates also an operator of ‘intention’ to represent different levels of culpability. Finally, [25] investigates the so-called T-STIT logic. T-STIT logic extends STIT logic with future and past tense operators, and with a group agency operator for coalitions involving all agents. The obtained logic is used to model normative concepts such as achievement obligation and commitment (see [22, Ch. 7], for details).

The logical frameworks mentioned above are highly expressive. This should not come as surprise, as these logics combine the expressivity of several operators of very different nature. In spite of this fact, the formalization of a particular deontic concept uses just a small fragment of these logics featuring a high expressive power, while such an expressivity impacts negatively on the overall computational behaviour of the logic. For instance, the work reported in [3, 20] studies fragments of STIT logic whose satisfiability problem is undecidable. The work from [27, 28] identifies fragments with complexities in ExpTime and NExpTime. Interestingly, in the last work it is also proven that very restricted fragments of STIT logics (i.e., those obtained by removing the temporal dimension or by limiting nested negations) are decidable in NP.

In this article, we take a different approach. We chose a specific, important deontic concept involving strategic reasoning: the notion of *knowingly complying*, and define a specialized formalism to work with it. This enables us to obtain a very natural representation while maintaining an excellent computational behaviour. Arguably, the

resulting logic is also very simple (and elegant) and leads itself well to a detailed study using already known techniques from modal logic [7, 9]. Our proposal takes inspiration from formalisms recently introduced in the epistemic study of the notion of *knowing how* (see, e.g., [2, 19, 32, 33]).

Contributions. We focus on representing the notion of *knowingly complying* in a deontic setting. We say that “*an agent knowingly complies with a certain goal φ , given a certain initial condition ψ* ”, whenever the agent has a normative course of action that leads from every situation in which ψ holds, only to situations where φ holds. Moreover, the agent tells apart such courses of actions from others that are not between the limits of the law. In doing so, we detect three main ingredients to consider (the terminology used here is inspired by [6]). First, we have the agents’ *abilities*, i.e., an account of what the agents are *able to do* (all the possible strategies or courses of action that an agent may take for achieving a goal). Second, we consider a set of *norms*, fixing what the agents *must* do. Norms are expressed as a set of legal courses of actions. Third, we take into account the *responsibilities* of the agents, given by each agent’s *own judgement*. This enables us to determine each agent’s awareness when complying with a norm or not. Remarkably, our framework does not rely on putting together separate existing features, and then coming up with a clever formalization. Instead, we will provide a semantics in which the above-mentioned concepts are internalized in the logic, and where each component interacts with the others to obtain the intended behaviour. This makes our framework different from existing approaches, instead of an alternative to them.

From a semantic perspective, we borrow and adapt ideas from the *epistemic knowing how logic* presented in [2]. Therein, an agent’s ‘know how’ is interpreted over *labelled transitions systems (LTS)*, extended with a notion of ‘epistemic indistinguishability’ at the level of plans (understood as finite sequences of basic actions). The relational part of the LTS provides the *ontic* information, i.e., the factual information that is common for all agents. The indistinguishability relation between plans provides instead the *epistemic* information for each agent, via their own perception about the real world. This approach is in line with standard epistemic logics [14, 21]. Here, we generalize this semantics, taking the LTS as the component representing the abilities available for all agents; a set of plans which establishes the set of normed courses of actions; and finally, an indistinguishability relation between plans that captures each agent’s own perception about the actions she can take. Then, we introduce corresponding modalities that enable us to express deontic properties over this kind of models: the deontic modality $N(\psi, \varphi)$ stating that “*in any situation in which ψ holds, it is possible to achieve the goal φ according to the norms*”; and the modality $Kc_i(\psi, \varphi)$, for each agent i , that states that “*in any situation in which ψ holds, agent i knowingly complies with φ* ”. We investigate the logical properties of this deontic logic, in particular we provide a sound and strongly complete axiom system and show that its satisfiability problem is NP-complete. Then, we extend the language with a modality S that enables us to express properties about the general abilities for the agents, and its interactions with the previously introduced modalities. Capturing abilities in the language enables us to distinguish between what agents can do vs. what agents do according to norms. For this extension we also provide an axiomatization.

EMERGENCY PROCEDURE	
– FIRE	KEEP CALM PULL FIRE ALARM, FROM A SAFE LOCATION CALL 999 (FIRE BRIGADE)
– SMOKE	
– EXPLOSION	
WHEN ALARM RINGS SHUT OFF GAS AND POWER	
Evacuate: close doors behind, use only stairs or ramps.	
If unsafe to evacuate: shut door, block cracks, stay low near window.	
If room door hot: keep door closed, stay put, stay low near window.	

Figure 1: Fire Emergency Evacuation Plan

Outline. In Sec. 2 we introduce an example that illustrates the choice of the logical components of our work. In Sec. 3 we define the syntax and semantics of the logic of *knowingly complying*. We accompany our definitions with the formal counterpart of the example from Sec. 2. In Sec. 4 we introduce a sound and complete axiom system, whereas in Sec. 5 we provide a characterization of the complexity of deciding satisfiability. In Sec. 6 we extend the logic in Sec. 3 with a new modality for capturing abilities in the language, and provide an axiomatization for this logic. Lastly, in Sec. 7 we provide some final remarks.

2 A MOTIVATING EXAMPLE

We motivate the logical components of our work using a fire emergency evacuation plan (FEEP) as a running example. A FEEP is a written document detailing the actions to be taken in the event of a fire, and the arrangements for calling the fire brigade. Fig. 1 illustrates a typical FEEP.¹ In spite of its simplicity, this FEEP contains some important norms to be complied with. For example, in the event of a fire/smoke/explosion, it is the duty of every person to take the following course of action: sound the nearest fire alarm, move to a safe location, and call the Fire Brigade. Moreover, every person should arrange for evacuating the premises in the light of some basic risk assessment and other emergency precautions, e.g., closing doors behind, or staying put and blocking doors if they are hot. The FEEP assumes awareness of the layout of the premises, capacity of identifying key escape routes via exit signs, and the possibility of vacating the premises using only stairs and/or ramps (elevators are excluded due to potential electrical failure and/or power outage). Finally, the procedure establishes that every person must remain calm in any possible situation, even if they have taken a wrong course of action (e.g., take an elevator in case of a fire).

In order to formalize how certain agents would act in a case of a fire, we detect the following basic components:

abilities: account for what the agents are ‘able to do’ to achieve a goal. In the FEEP, for instance, agents are able to take the stairs to bring themselves from an unsafe to a safe place. Certain combinations should also be possible, e.g., the FEEP establishes that in a case of fire, the agents need to “pull the alarm, take stairs/ramps to evacuate, and finally call 999”. These sequences, commonly called *plans*, is what we understand as a course of action. In our setting, these abilities will be characterized via *Labelled Transition Systems (LTSs)*.

¹In fact, Fig. 1 takes inspiration from the FEEP obtainable from the Occupational Safety section on McMaster University’s website.

norms: account for ‘legal’ courses of actions. They reflect the fact that some courses of action in some situations are regulated by norms. In our FEEP example, the plan “pull the alarm, take the stairs (or the ramp) and call 999” is prescribed in the case of a fire, whereas “use the elevator” is preempted.

responsibilities: captures each agent’s own judgement for complying with a norm. From the perspective of a particular agent, there might be certain courses of action that are indistinguishable from others (equivalent among themselves with respect to the current scenario). E.g., agent i may consider that exiting the building using stairs or ramps is fine, while evacuating the premises using elevators should be avoided (as indicated by the FEEP). While agent j , that ignores the FEEP, may take that exiting the premises, no matter how, is what matters. In our formal setting, we will want to express that agent i knowingly complies with the norms, whereas j does not. Moreover, some agents might even be unaware of certain courses of action being possible; e.g., the location of a particular exit point. To do so, we will define a suitable notion of “being indistinguishable” between plans.

Notice that each component, uses and refines the previous ones. This will become clearer in the next section. In the rest of the paper we develop the formal machinery behind the provided intuitions.

3 DEONTIC LOGIC OF KNOWINGLY COMPLYING

In this section, we introduce the language and the semantics of our Deontic Logic of Knowingly Complying (DLKc). We assume denumerable sets Prop for proposition symbols, and Act for basic action names, and that Agt is a non-empty finite set of agent names. Moreover, we assume all these sets are pairwise disjoint.

DEFINITION 3.1. The language of DLKc, i.e., its set of formulas, is determined by the following grammar:

$$\varphi, \psi ::= p \mid \neg\varphi \mid \varphi \vee \psi \mid N(\psi, \varphi) \mid Kc_i(\psi, \varphi),$$

where: $p \in \text{Prop}$ and $i \in \text{Agt}$. We use $\perp, \top, \varphi \wedge \psi, \varphi \rightarrow \psi$, and $\varphi \leftrightarrow \psi$ as abbreviations defined as usual. Intuitively, a formula $N(\psi, \varphi)$ is read as: “there is a normative course of action that brings about φ given ψ ”; and a formula $Kc_i(\psi, \varphi)$ is read as: “agent i knowingly complies with φ given ψ ”. We also write $A\varphi$ and read it as: “ φ holds anywhere”; and $E\varphi$ read it as: “ φ holds somewhere”. The connectives A and E are the universal and existential modalities [17]. As shown in Prop. 3.1, they are definable in terms of other connectives.

EXAMPLE 3.1. Tab. 1 illustrates how to use formulas of DLKc to express some properties in the context of the example in Sec. 2.

We introduce the structures on which to interpret the formulas of DLKc in a step by step fashion.

DEFINITION 3.2 (PLANS). We denote Act^* the set of all (possibly-empty) finite sequences over Act. The elements of Act^* are called plans. We use ϵ to denote the empty sequence in Act^* (the empty plan). For $\pi \in \text{Act}^*$, $|\pi|$ is the length of π . For $0 \leq k \leq |\pi|$, π_k is the initial segment of π of length k and $\pi[k]$ is the k^{th} element of π .

Intuitively, the elements in Act can be understood in correspondence to basic actions, and those in Act^* in correspondence to courses of action.

Table 1: Formulas and their Intuitive Readings.

Formula	Intuitive reading
$A(s \rightarrow \neg f)$	In any safe location (s), there is no fire (f).
Ef	There is the possibility of a fire.
$N(f \wedge c, s)$	There exists a normative course of action that brings any agent to a safe location in case of a fire, whenever there is the capacity (c) to do so.
$Kc_i(f \wedge c, s)$	Agent i knowingly complies with the norms regulating reaching a safe location in the event of a fire (a.k.a., knows how to conform to the FEEP), provided also that there is a capacity (c) to do so.

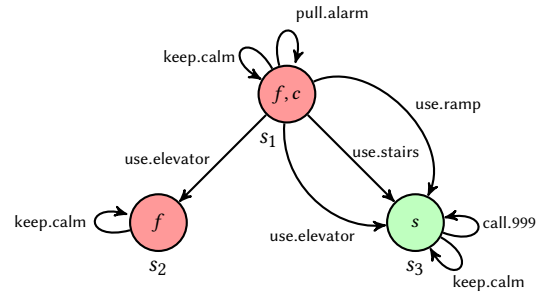


Figure 2: An LTS for the FEEP.

EXAMPLE 3.2. For the FEEP in Sec. 2, we may have basic action names such as:

keep.calm, pull.alarm, call.999,
use.stairs, use.ramp, use.elevator, ...

These basic actions names represent the actions of: remaining calm, pulling the fire alarm; calling the Fire Brigade; and using the stairs, a ramp, and an elevator to exit the building, resp. In turn, the plan

$$\pi_1 = \text{pull.alarm}; \text{use.stairs}; \text{call.999}$$

indicates the course of action of pulling the fire alarm, evacuating the building using the stairs, and calling the Fire Brigade; while

$$\pi_2 = \text{use.elevator}$$

would indicate the plan of using an elevator to exit the building.

In general, we will not be interested in all plans, but only on those that are somewhat delimited (i.e., we would like to rule out arbitrary arrangements of basic actions). Moreover, we may wish for certain plans to take place only in particular contexts, and with the purpose of bringing about some goal. These ideas are made precise in Def. 3.3.

DEFINITION 3.3 (LTS). A labelled transition system (LTS) is a tuple $\mathcal{Q} = \langle S, R, V \rangle$ where: S is a non-empty set of states; for some $A \subseteq \text{Act}$, R is an A -indexed family of binary relations on S , i.e., $R = \{R_a \subseteq S^2 \mid a \in A\}$; and $V : S \rightarrow 2^{\text{Prop}}$ is an assignment function.

Simply put, \mathcal{Q} is a graph whose nodes are labelled with proposition symbols and whose edges are labelled with action names. In this way, V indicates the proposition symbols that hold on each state; whereas R represents –following the terminology in [32]– an ability map, i.e., the possible courses of action for all agents.

Table 2: Intuitive Interpretation of the states in Fig. 2.

state	represents a situation in which:
s_1	A fire occurs, and there is the capacity to follow the evacuation protocol.
s_2	A fire occurs, but there is no capacity to follow the evacuation protocol (e.g., trapped in elevator).
s_3	A safe location has been reached, thus there is no fire.

EXAMPLE 3.3. In Fig. 2 we present an LTS modelling a part of the FEEP in Sec. 2. The intuitive interpretation of the states in this LTS is summarized in Tab. 2.

Notice how the LTS adds contexts/goals (pre/post conditions) to basic actions, and thus to plans. We summarize what these contexts/goals look like for the plans in Ex. 3.2 in the table below.

plan	pre	post	plan	pre	post
π_1	$f \wedge c$	s	π_2	$f \wedge c$	$(f \wedge \neg c) \vee (s \wedge \neg f)$

Intuitively, the plan π_1 has as a context the occurrence of a fire and the capacity to follow the evacuation protocol, i.e., it is a fact that such a capacity exists; its goal is that of taking the agent to a safe place (and thus one in which there is no fire). In turn, plan π_2 takes the agent to a situation in which there is still a fire, and in which it has lost the capacity to follow the evacuation protocol and has not reached a safe location (state s_2); or to a state in which it has reached a safe location (s_3). This non-determinism in π_2 captures the potential failure of the elevator as a mean for evacuation.

Defs. 3.4 and 3.5 make precise the possible plans in an LTS.

DEFINITION 3.4. Let R and R' be binary relations on a set S . For $S' \subseteq S$; $R(S') = \{s \in S \mid s' \in S' \text{ and } (s', s) \in R\}$; we write $R(\{s\})$ instead of $R(\{s\})$. The (sequential) composition of R and R' is defined as $RR' = \{(s_1, s_2) \mid \text{exists } s \in S \text{ s.t. } (s_1, s) \in R \text{ and } (s, s_2) \in R'\}$. In turn, let $\mathfrak{L} = \langle S, R, V \rangle$ be an LTS (with R defined over $A \subseteq \text{Act}$), and let $\pi \in \text{Act}^*$ be such that $|\pi| = n$;

$$R_\pi = \begin{cases} \{(s, s) \mid s \in S\} & \text{if } \pi = \epsilon \\ \emptyset & \text{if exists } 0 < k \leq n \text{ s.t. } \pi[k] \notin A \\ R_{\pi[1]} \dots R_{\pi[n]} & \text{otherwise.} \end{cases}$$

For $\Pi \subseteq \text{Act}^*$; $R_\Pi = \bigcup \{R_\pi \mid \pi \in \Pi\}$.

DEFINITION 3.5 (STRONG EXECUTABILITY). Let $\mathfrak{L} = \langle S, R, V \rangle$ be an LTS, $s \in S$, and $\pi \in \text{Act}^*$; we say that π is strongly executable (SE) at s iff for all $0 \leq k \leq |\pi| - 1$ and all $s' \in R_{\pi_k}(s)$, $R_{\pi[k+1]}(s') \neq \emptyset$. The set of all states in which π is SE is $\text{SE}(\pi) = \{s \in S \mid \pi \text{ is SE at } s\}$. $\Pi \subseteq \text{Act}^*$ is strongly executable at s iff for every $\pi \in \Pi$, $s \in \text{SE}(\pi)$. The set of all states in which Π is SE is $\text{SE}(\Pi) = \bigcap \{\text{SE}(\pi) \mid \pi \in \Pi\}$.

The notion of strong executability in Def. 3.5 states that a plan is fail proof, i.e., each time a plan commences at some state, it carries through. This technical requirement is inspired by conformant planning [29], and justified at a conceptual level in [32–34].

EXAMPLE 3.4. Let \mathfrak{L} be the LTS in Fig. 2; the plans π_1 and π_2 in Ex. 3.2 are SE at s_1 . The plan $\pi_3 = \text{keep.calm}$ is SE everywhere. It is easy to see that the plan

$$\pi_4 = \text{pull.alarm; use.elevator; call.999}$$

is not SE at s_1 : if we take pull.alarm and then use.elevator, we may land in state s_2 , where it is not possible to take call.999.

Thus far the picture is fairly standard. We do, however, enrich our models with a normative component.

DEFINITION 3.6 (NORMATIVE LTS). A normative LTS (NLTS) is a tuple $\mathfrak{N} = \langle S, R, V, N \rangle$ where: $\mathfrak{L} = \langle S, R, V \rangle$ is an LTS, and $N \subseteq \text{Act}^*$ is a set of plans such that there is $\pi \in N$ satisfying the condition $\text{SE}(\pi) = S$. We refer to N as the set of normative plans of \mathfrak{N} .

The set N in an NLTS is intuitively understood as a set of normative plans. The requirement on N having at least one strongly executable plan can be understood as “there are norms that can always be complied with”. This takes inspiration from the deontic property “ought” implies “can” (or seriality in modal logic).

EXAMPLE 3.5. Continuing with the FEEP example in Sec. 2, it would be reasonable to have: $N = \{\pi_1, \pi_3, \pi_5\}$, where π_1 and π_3 are as in Exs. 3.2 and 3.4, respectively, and $\pi_5 = \text{pull.alarm; use.ramp; call.999}$. The set N captures the normative aspects of the FEEP which dictate that in case of a fire an agent shall evacuate the building using only stairs or ramps. Notice the occurrence of $\pi_3 = \text{keep.calm}$ in N . This is an indication that in the case of an emergency we should always remain calm. The plan π_3 also guarantees the model is a normative LTS, as it is SE everywhere in the model (Def. 3.6).

Our final ingredient is the perception each agent has of a given scenario. The main point to be made is that agents have their own awareness of the courses of action they can take, with some being indistinguishable. This incorporates ideas from [2].

DEFINITION 3.7 (U-NLTS). An uncertainty-based normative LTSs (U-NLTS) is a tuple $\mathfrak{M} = \langle S, R, V, U, N \rangle$ where: $\mathfrak{N} = \langle S, R, V, N \rangle$ is an NLTS; and $U : \text{Agt} \rightarrow 2^{2^{\text{Act}^*}}$ satisfies: $\emptyset \in U(i)$, and for all $\{\Pi, \Pi'\} \subseteq U(i)$, $\Pi \neq \Pi'$ implies $\Pi \cap \Pi' = \emptyset$. If $s \in S$, the pair (\mathfrak{M}, s) is a pointed U-NLTS (parentheses usually dropped).

Intuitively, the function U in an U-NLTS indicates courses of actions that are indistinguishable from the perspective of an agent. More precisely, each $\Pi \in U(i)$ captures the courses of actions that are, from the perspective of agent i , as good as any other in this set. The condition $\emptyset \in U(i)$ indicates that the ‘abort’ plan (i.e., a plan that always fails) is possible, and that agents can tell it apart from the rest of plans. Note that $\Pi_i = \bigcup \{\Pi \mid \Pi \in U(i)\}$ assigns a set of plans to an agent i . The set Π_i captures a sense of awareness for agent i , i.e., it tells what courses of action this agent may engage on. In this way, U gives rise to an equivalence relation over each Π_i , similar to the standard Epistemic Logic, but at the level of plans.

EXAMPLE 3.6. Adding to the example in Sec. 2, let us suppose that we have agents i and j , and that i has taken an occupational safety course, but j has not. Then, i should know the difference between using stairs/ramps and using the elevator to evacuate the building in case of a fire. On the other hand, for j all possible ways of exiting the building might be equally good. In this setting,

$$U(i) = \{\{\pi_1, \pi_5\}, \{\pi_4\}\} \quad U(j) = \{\{\pi_1, \pi_4, \pi_5\}\},$$

where π_1 , π_4 and π_5 are as in Exs. 3.2, 3.4 and 3.5, respectively.

At this point we have all the ingredients that are necessary to introduce the formal semantics of our logic.

DEFINITION 3.8 (SEMANTICS). Let $\mathfrak{M} = \langle S, R, V, U, N \rangle$ be a U-NLTS, $s \in S$, and φ be a formula; $\mathfrak{M}, s \Vdash \varphi$ is defined as:

$$\begin{aligned} \mathfrak{M}, s \Vdash p & \text{ iff } p \in V(s), \\ \mathfrak{M}, s \Vdash \neg\varphi & \text{ iff } \mathfrak{M}, s \not\Vdash \varphi, \\ \mathfrak{M}, s \Vdash \varphi \vee \psi & \text{ iff } \mathfrak{M}, s \Vdash \varphi \text{ or } \mathfrak{M}, s \Vdash \psi, \\ \mathfrak{M}, s \Vdash N(\psi, \varphi) & \text{ iff exists } \pi \in N \text{ such that} \\ & \text{(i) } \llbracket \psi \rrbracket^{\mathfrak{M}} \subseteq SE(\pi) \text{ and} \\ & \text{(ii) } R_{\pi}(\llbracket \psi \rrbracket^{\mathfrak{M}}) \subseteq \llbracket \varphi \rrbracket^{\mathfrak{M}}, \\ \mathfrak{M}, s \Vdash Kc_i(\psi, \varphi) & \text{ iff exists } \Pi \in U(i) \text{ such that} \\ & \text{(i) } \Pi \subseteq N, \\ & \text{(ii) } \llbracket \psi \rrbracket^{\mathfrak{M}} \subseteq SE(\Pi), \text{ and} \\ & \text{(iii) } R_{\Pi}(\llbracket \psi \rrbracket^{\mathfrak{M}}) \subseteq \llbracket \varphi \rrbracket^{\mathfrak{M}}, \end{aligned}$$

where $\llbracket \chi \rrbracket^{\mathfrak{M}} = \{s \in S \mid \mathfrak{M}, s \Vdash \chi\}$.

Intuitively, $N(\psi, \varphi)$ states that there is a normative plan to bring about φ given ψ (i.e., a plan that is supported by the norms in N). In turn, $Kc_i(\psi, \varphi)$ states that there is a set of normative plans, all indistinguishable from the perspective of agent i , each of which brings about φ given ψ . The normative reading of the Kc_i operator is that, the agent knows how to achieve φ given ψ within the limits of some norms. Thus, we refer to Kc_i as *knowingly complying*.

Let us now turn our attention onto how to define the universal and existential modalities. Let $A\varphi = N(\neg\varphi, \perp)$ and $E\varphi = \neg A\neg\varphi$. Prop. 3.1 shows that these definitions indeed capture the usual reading of these modalities. The proof of this proposition relies on the fact that the set N is never empty and contains some π such that $SE(\pi) = S$, following closely the argument for the *knowing* how operator Kh in [32].

PROPOSITION 3.1. Let $\mathfrak{M} = \langle S, R, V, U, N \rangle$ be a U-NLTS, $s \in S$, and φ be a formula; $\mathfrak{M}, s \Vdash A\varphi$ iff $\llbracket \varphi \rrbracket^{\mathfrak{M}} = S$.

Prop. 3.2 tells us that the modalities being considered are *global*, i.e., they hold anywhere or nowhere in the model.

PROPOSITION 3.2. It holds that $\mathfrak{M}, s \Vdash N(\psi, \varphi)$ iff $\mathfrak{M}, s \Vdash AN(\psi, \varphi)$; and $\mathfrak{M}, s \Vdash Kc_i(\psi, \varphi)$ iff $\mathfrak{M}, s \Vdash AKc_i(\psi, \varphi)$.

We conclude this section by summarizing our running example.

EXAMPLE 3.7. Let \mathfrak{M} be the U-NLTS composed of the parts detailed in Exs. 3.3, 3.5 and 3.6; it is easy to show that:

$$\begin{aligned} (1) \mathfrak{M}, s_1 \Vdash A(s \rightarrow \neg f) & \quad (3) \mathfrak{M}, s_1 \Vdash N(f \wedge c, s) \\ (2) \mathfrak{M}, s_1 \Vdash Ef & \quad (4) \mathfrak{M}, s_1 \Vdash Kc_i(f \wedge c, s) \\ & \quad (5) \mathfrak{M}, s_1 \not\Vdash Kc_j(f \wedge c, s) \end{aligned}$$

(1) and (2) are immediate. As a witness for (3) we can take, e.g. the plan π_1 in Ex. 3.2. As a witness for (4) we can take, e.g. the set $\{\pi_1, \pi_5\} \in U(i)$. Failure of (5) obtains from the fact that $\{\pi_1, \pi_4, \pi_5\} \in U(j)$ and that $\{\pi_1, \pi_4, \pi_5\} \not\subseteq N$.

4 AXIOM SYSTEM

In this section, we present a sound and complete axiom system (see Tab. 3) for DLKc. It comes to light immediately that the universal and the existential modalities A and E (definable in DLKc) are instrumental in these results.

The soundness of the axiom system \mathcal{DLKc} in Tab. 3 is direct. Before establishing its completeness (Thm. 1), we offer some comments. We start with the first block of axioms. The axiomatization

Table 3: Axiom system \mathcal{DLKc} for DLKc over U-NLTSs.

Axioms:	
Taut	$\vdash \varphi$ for φ a propositional tautology
DistA	$\vdash A(\psi \rightarrow \varphi) \rightarrow (A\psi \rightarrow A\varphi)$
TA	$\vdash A\varphi \rightarrow \varphi$
4KcA	$\vdash Kc_i(\psi, \varphi) \rightarrow AKc_i(\psi, \varphi)$
5KcA	$\vdash \neg Kc_i(\psi, \varphi) \rightarrow A\neg Kc_i(\psi, \varphi)$
4NA	$\vdash N(\psi, \varphi) \rightarrow AN(\psi, \varphi)$
5NA	$\vdash \neg N(\psi, \varphi) \rightarrow A\neg N(\psi, \varphi)$
KcN	$\vdash Kc_i(\psi, \varphi) \rightarrow N(\psi, \varphi)$
DN	$\vdash N(\varphi, \top)$
KcA	$\vdash (A(\psi \rightarrow \chi) \wedge Kc_i(\chi, \rho) \wedge A(\rho \rightarrow \varphi)) \rightarrow Kc_i(\psi, \varphi)$
NA	$\vdash (A(\psi \rightarrow \chi) \wedge N(\chi, \rho) \wedge A(\rho \rightarrow \varphi)) \rightarrow N(\psi, \varphi)$
Kc \perp	$\vdash Kc_i(\perp, \perp)$
Rules:	
$\frac{\vdash \psi \quad \vdash (\psi \rightarrow \varphi)}{\vdash \varphi}$ (MP)	$\frac{\vdash \varphi}{\vdash A\varphi}$ (Nec)

of the universal modality A needs only normality (given by axiom DistA and the rule Nec) and reflexivity (TA). As shown in [32], symmetry and transitivity for A are theorems of the system, since they can be derived from particular instances of axioms 4NA and 5NA (discussed below). The second block of axioms, 4KcA to 5NA, captures the fact that the two modalities of the language are global (see Prop. 3.2). Lastly, we turn our attention to the third block of axioms. Here, we point out that axiom KcN fixes the interaction between the two modalities, whereas DN establishes a form of *seriality* for the deontic modality N . Intuitively, DN states that, from any situation, there is always a legal way to achieve an universally valid goal (\top). Of the remaining axioms, KcA and NA, state that Kc_i and N , respectively, are closed under global entailment; whereas Kc \perp tells us how agents behave in impossible situations.

PROPOSITION 4.1. The following formulas are derivable using the axioms and rules in Tab. 3:

$$KcE \quad (E\psi \wedge Kc_i(\psi, \varphi)) \rightarrow E\varphi; \quad NE \quad (E\psi \wedge N(\psi, \varphi)) \rightarrow E\varphi$$

At this point, we turn our attention to completeness. We begin with some preliminary definitions.

DEFINITION 4.1. Let Φ be the set of all maximally consistent sets (MCS) of formulas (w.r.t. \mathcal{DLKc}); for any $\Gamma \in \Phi$, define:

$$\begin{aligned} \Gamma|_N &= \{N(\psi, \varphi) \mid N(\psi, \varphi) \in \Gamma\} & \Gamma|_A &= \{A\varphi \mid A\varphi \in \Gamma\} \\ \Gamma|_{Kc_i} &= \{Kc_i(\psi, \varphi) \mid Kc_i(\psi, \varphi) \in \Gamma\} & \Gamma|_{Kc} &= \bigcup \{\Gamma|_{Kc_i} \mid i \in \text{Agt}\}. \end{aligned}$$

For $\Gamma \in \Phi$; define $\text{Act}^\Gamma = \{\langle \psi, \varphi \rangle \mid N(\psi, \varphi) \in \Gamma\}$.

REMARK 4.1. Note that Act^Γ is denumerable, thus it is an adequate set of actions for building a model. Note also that Act^Γ fixes a new signature. This causes no problem since the operators of the language cannot see the names of the actions; i.e., we can define a mapping from Act^Γ to any particular Act and preserve the original signature.

Let us take the first step towards an adequate notion of canonical model \mathfrak{M}^Γ for an MCS $\Gamma \in \Phi$. Using standard ideas from modal logic [7], we would take the set S of states of \mathfrak{M}^Γ to be Φ . In doing this, however, we run into a problem. We may have, $Kc_i(\psi, \varphi) \in s$ and $\neg Kc_i(\psi, \varphi) \in s'$, for some $\{s, s'\} \subseteq S$ (since $S = \Phi$ contains all MCSs w.r.t. \mathcal{DLKc}). This causes the *Truth-Lemma* to fail: it should happen that $\mathfrak{M}^\Gamma, s \Vdash Kc_i(\psi, \varphi)$ iff for all $s'' \in S$, $\mathfrak{M}^\Gamma, s'' \Vdash Kc_i(\psi, \varphi)$ (see Prop. 3.2). Yet, we have s' such that $\mathfrak{M}^\Gamma, s' \Vdash \neg Kc_i(\psi, \varphi)$, and so

\mathfrak{M}^Γ , $s' \not\models \text{Kc}_i(\psi, \varphi)$, a contradiction. Moreover, a similar argument can be put forth for N .

The scenario described above tells us that we need to do some extra work in building our canonical model, similar to what happens in e.g., [17] and in [18] for the universal modality and Propositional Dynamic Logic (PDL), respectively. In those cases, the corresponding structure needs to satisfy some additional constraint. In the former, the situation is similar than ours, as the canonical model is generated by the modality A . For the latter, the canonical model is filtrated in order to characterize the transitive closure of a relation.

Simply put, we need to select the appropriate set of MCS in order to fulfill the ‘globality’ requirement for our modal formulas, established in the block of axioms 4KcA to 5NA . This extra work is made precise in Def. 4.2.

DEFINITION 4.2. *The canonical model of an MCS of formulas $\Gamma \in \Phi$ is a tuple $\mathfrak{M}_c^\Gamma = \langle S^\Gamma, R^\Gamma, V^\Gamma, U^\Gamma, N^\Gamma \rangle$ where:*

$$\begin{aligned} S^\Gamma &= \{\Delta \in \Phi \mid \Delta|_{\text{A}} = \Gamma|_{\text{A}}\} \\ R_{\langle \psi, \varphi \rangle}^\Gamma &= \{(\Delta_1, \Delta_2) \in S^\Gamma \times S^\Gamma \mid \text{N}(\psi, \varphi) \in \Gamma, \psi \in \Delta_1, \varphi \in \Delta_2\} \\ R^\Gamma &= \{R_{\langle \psi, \varphi \rangle}^\Gamma \mid \text{N}(\psi, \varphi) \in \Gamma\} \\ V^\Gamma(\Delta) &= \{p \in \text{Prop} \mid p \in \Delta\} \\ U^\Gamma(i) &= \{\{\langle \psi, \varphi \rangle\} \mid \text{Kc}_i(\psi, \varphi) \in \Gamma\} \cup \{\emptyset\} \\ N^\Gamma &= \{\langle \psi, \varphi \rangle \mid \text{N}(\psi, \varphi) \in \Gamma\}. \end{aligned}$$

Notice that \mathfrak{M}_c^Γ is generated by formulas of the form $\text{A}\varphi$; i.e., the (global) formulas that occur in Γ .

PROPOSITION 4.2. *The following are immediate for \mathfrak{M}_c^Γ :*

- (1) $\{\langle \psi, \varphi \rangle\} \in U^\Gamma(i)$ implies $\langle \psi, \varphi \rangle \in N^\Gamma$;
- (2) $\langle \psi, \varphi \rangle \in N^\Gamma$ iff $\langle \psi, \varphi \rangle \in \text{Act}^\Gamma$;
- (3) $R_{\langle \psi, \varphi \rangle}^\Gamma \neq \emptyset$ implies $\text{N}(\psi, \varphi) \in \Gamma$.

Items (1) of Prop. 4.2 follows by axiom KcN . Items (2) and (3) follow by definition of \mathfrak{M}_c^Γ . Now, we need to show that \mathfrak{M}_c^Γ is a proper U-NLTS.

PROPOSITION 4.3. *$\mathfrak{M}_c^\Gamma = \langle S^\Gamma, R^\Gamma, V^\Gamma, U^\Gamma, N^\Gamma \rangle$ is a U-NLTS.*

PROOF. It is clear that $\langle S^\Gamma, R^\Gamma, V^\Gamma \rangle$ is an LTS (e.g., $S^\Gamma \neq \emptyset$, as $\Gamma \in S^\Gamma$). Then, we need to show that there exists $\pi \in N^\Gamma$ s.t. $\text{SE}(\pi) = S^\Gamma$, as per Def. 3.6. Notice that $\text{N}(\varphi, \top) \in \Gamma$ for every $\varphi \in \text{DLKc}$ (by DN). In particular, $\text{N}(\top, \top) \in \Gamma$. Hence, $\langle \top, \top \rangle \in N^\Gamma$, and $\text{SE}(\langle \top, \top \rangle) = S^\Gamma$.

It remains to show that U^Γ satisfies the conditions of Def. 3.7. By definition, $\emptyset \in U^\Gamma(i)$. Let $\Pi_1, \Pi_2 \in U_i^\Gamma - \{\emptyset\}$ be s.t. $\Pi_1 = \{\langle \psi_1, \varphi_1 \rangle\} \neq \{\langle \psi_2, \varphi_2 \rangle\} = \Pi_2$ (recall that Π_1, Π_2 are singletons). Then, $\Pi_1 \cap \Pi_2 = \emptyset$. \square

Below we state some properties of \mathfrak{M}_c^Γ , that will be useful in what follows. We start with a property about the global relations between the states of \mathfrak{M}_c^Γ , whose proof relies on the axioms of the block 4KcA to 5NA .

PROPOSITION 4.4. *Let $\{\Delta_1, \Delta_2\} \subseteq S^\Gamma$ and $X \in \{\text{Kc}_i, \text{N}\}$; we have $\Delta_1|_X = \Delta_2|_X = \Gamma|_X$.*

Next, we establish some properties about the structure of \mathfrak{M}_c^Γ (see [2, 34] for details).

PROPOSITION 4.5. *If $R_{\langle \psi, \varphi \rangle}^\Gamma(\Delta) \neq \emptyset$, then, for all $\Delta' \in S^\Gamma$, $\varphi \in \Delta'$ implies $\Delta' \in R_{\langle \psi, \varphi \rangle}^\Gamma(\Delta)$.*

PROPOSITION 4.6. *For any formula φ ; if $\varphi \in \Delta$ for every $\Delta \in S^\Gamma$, then $\text{A}\varphi \in \Delta$ for every $\Delta \in S^\Gamma$.*

PROPOSITION 4.7. *If $\psi \in \Delta$ then, $R_{\langle \psi, \varphi' \rangle}^\Gamma(\Delta) \neq \emptyset$, implies for all $\Delta' \in S^\Gamma$, $\text{A}(\psi \rightarrow \psi') \in \Delta'$.*

PROPOSITION 4.8. *Let $X \in \{\text{Kc}_i, \text{N}\}$; if there is $\Theta \in S^\Gamma$ such that $\{\psi, X(\psi, \varphi)\} \subseteq \Theta$, then there is $\Theta' \in S^\Gamma$ such that $\varphi \in \Theta'$.*

Props. 4.4 to 4.8 are instrumental in our proof of the Truth Lemma for \mathfrak{M}_c^Γ , which is stated below.

LEMMA 4.1 (TRUTH LEMMA). *Let $\Gamma \in \Phi$, and let \mathfrak{M}_c^Γ be as in Def. 4.2; for all $\Delta \in S^\Gamma$, and for all $\varphi \in \text{DLKc}$, $\mathfrak{M}_c^\Gamma, \Delta \models \varphi$ iff $\varphi \in \Delta$.*

PROOF. Let $\mathfrak{M}_c^\Gamma = \langle S^\Gamma, R^\Gamma, V^\Gamma, U^\Gamma, N^\Gamma \rangle$. The proof is by induction on the structure of φ . The atomic and Boolean cases are as usual, so we focus on the Kc_i case (the case of N being similar).

Case $\varphi = \text{Kc}_i(\psi, \rho)$: (\Rightarrow) Suppose $\mathfrak{M}_c^\Gamma, \Theta \models \text{Kc}_i(\psi, \rho)$. Then, there is $\Pi \in U^\Gamma(i)$ such that: $\Pi \subseteq N^\Gamma$, $\llbracket \psi \rrbracket^{\mathfrak{M}_c^\Gamma} \subseteq \text{SE}(\Pi)$ and $R_\Pi(\llbracket \psi \rrbracket^{\mathfrak{M}_c^\Gamma}) \subseteq \llbracket \rho \rrbracket^{\mathfrak{M}_c^\Gamma}$. We have two cases:

- If $\llbracket \psi \rrbracket^{\mathfrak{M}_c^\Gamma} = \emptyset$, then by IH, $\neg\psi \in \Theta'$ for all $\Theta' \in S^\Gamma$. Thus, by Prop. 4.6, $\text{A}\neg\psi \in \Theta'$ for all $\Theta' \in S^\Gamma$, and therefore, $\text{A}(\psi \rightarrow \perp) \in \Theta'$. Since $\perp \rightarrow \rho$ is a tautology, by Nec, $\text{A}(\perp \rightarrow \rho) \in \Theta'$. Using an instance of KcA , $(\text{A}(\psi \rightarrow \perp) \wedge \text{Kc}_i(\perp, \perp) \wedge \text{A}(\perp \rightarrow \rho)) \rightarrow \text{Kc}_i(\psi, \rho) \in \Theta'$. Then, by $\text{Kc}\perp$ and MP, we get that $\text{Kc}_i(\psi, \rho) \in \Theta'$, for all $\Theta' \in S^\Gamma$. Thus, $\text{Kc}_i(\psi, \rho) \in \Theta$.
- If $\llbracket \psi \rrbracket^{\mathfrak{M}_c^\Gamma} \neq \emptyset$, then $\Pi = \{\langle \psi', \varphi' \rangle\}$ since $\Pi = \emptyset$ would force $\llbracket \psi \rrbracket^{\mathfrak{M}_c^\Gamma} = \emptyset$. Take $\Theta \in S^\Gamma$ such that $\mathfrak{M}_c^\Gamma, \Theta \models \psi$. By IH, $\psi \in \Theta$. Moreover, for all $\Theta' \in S^\Gamma$, if $\psi \in \Theta'$ then:
 - Θ' has an $R_{\langle \psi', \varphi' \rangle}^\Gamma$ -successor, and
 - for all $\Theta'' \in S^\Gamma$ s.t. $(\Theta', \Theta'') \in R_{\langle \psi', \varphi' \rangle}^\Gamma$, $\rho \in \Theta''$ (IH).

Note that as Θ' has an $R_{\langle \psi', \varphi' \rangle}^\Gamma$ -successor, $\psi' \in \Theta$, and therefore, $\psi \rightarrow \psi'$, for all $\Theta' \in S^\Gamma$. By Prop. 4.7, $\text{A}(\psi \rightarrow \psi') \in \Theta'$ for all $\Theta' \in S^\Gamma$. By Prop. 4.5, every $\Theta'' \in S^\Gamma$ such that $\varphi' \in \Theta''$ can be $R_{\langle \psi', \varphi' \rangle}^\Gamma$ -reached from Θ' . Therefore, for every $\Theta' \in S^\Gamma$ such that $\psi \in \Theta'$ we have that every $\Theta'' \in S^\Gamma$ such that $\varphi' \in \Theta''$ can be $R_{\langle \psi', \varphi' \rangle}^\Gamma$ -reached from Θ' ; i.e., $\Theta'' \in \llbracket \varphi' \rrbracket^{\mathfrak{M}_c^\Gamma} \subseteq R_\Pi^\Gamma(\llbracket \psi \rrbracket^{\mathfrak{M}_c^\Gamma}) \subseteq \llbracket \rho \rrbracket^{\mathfrak{M}_c^\Gamma}$. Then, we get $\rho \in \Theta''$. Thus, for all $\Theta'' \in S^\Gamma$, $\varphi' \rightarrow \rho \in \Theta''$. Using Prop. 4.6, for all $\Theta'' \in S^\Gamma$, $\text{A}(\varphi' \rightarrow \rho) \in \Theta''$. Finally, putting all together, for all $\Theta' \in S^\Gamma$, $\{\text{A}(\psi \rightarrow \psi'), \text{Kc}_i(\psi', \varphi'), \text{A}(\varphi' \rightarrow \rho)\} \subseteq \Theta'$. By axiom KcA , $\text{Kc}_i(\psi, \rho) \in \Theta'$ and thus, $\text{Kc}_i(\psi, \rho) \in \Theta$.

(\Leftarrow) Suppose $\text{Kc}_i(\psi, \rho) \in \Theta$. Then, by Prop. 4.4, $\text{Kc}_i(\psi, \rho) \in \Theta'$ for all $\Theta' \in S^\Gamma$. Moreover, $\text{Kc}_i(\psi, \rho) \in \Gamma$ and $R_{\langle \psi, \rho \rangle}^\Gamma$ is defined. To prove that $\mathfrak{M}_c^\Gamma, \Theta \models \text{Kc}_i(\psi, \rho)$, we have to consider two cases:

- There is no Θ' such that $\psi \in \Theta'$. By IH, $\llbracket \psi \rrbracket^{\mathfrak{M}_c^\Gamma} = \emptyset$. Using $\Pi = \emptyset$, we trivially have that $\mathfrak{M}_c^\Gamma, \Theta \models \text{Kc}_i(\psi, \rho)$.
- There is Θ' such that $\psi \in \Theta'$: by Prop. 4.8, there is Θ'' such that $\rho \in \Theta''$. By IH, $\mathfrak{M}_c^\Gamma, \Theta' \models \psi$ and $\mathfrak{M}_c^\Gamma, \Theta'' \models \rho$. Since it is defined, $\Pi = \{\langle \psi, \rho \rangle\}$ is SE at all ψ -states (since there is an

$R_{\langle\psi,\rho\rangle}^\Gamma$ -successor Θ''), reaches from these only ρ -states via Π (by construction of $R_{\langle\psi,\rho\rangle}^\Gamma$), and $\{\langle\psi,\rho\rangle\} \subseteq N^\Gamma$ (by Prop. 4.2). Thus, $\mathfrak{M}_c^\Gamma, \Theta \Vdash Kc_i(\psi, \rho)$. \square

Following from Lemma 4.1, and using a standard argument, we establish the following result.

THEOREM 1. *The axiom system \mathcal{DLKc} in Tab. 3 is sound and strongly complete for DLKc over the class of all U-NLTSs.*

5 COMPLEXITY

In this section, we investigate the computational complexity of the satisfiability problem of DLKc. For this logic, we will establish membership in NP by showing a polynomial size model property.

Given a formula, we will show that it is possible to select just a piece of the canonical model which is relevant for its evaluation. The selected model will preserve satisfiability, and moreover, its size will be polynomial w.r.t. the size of the input formula.

DEFINITION 5.1 (SELECTION FUNCTION). *Let Γ be a MCS, and let $\mathfrak{M}_c^\Gamma = \langle S^\Gamma, R^\Gamma, V^\Gamma, U^\Gamma, N^\Gamma \rangle$ be a canonical model; let $w \in S^\Gamma$ and φ be a formula. Define $\text{Act}_\varphi = \{\langle\theta_1, \theta_2\rangle \in \text{Act}^\Gamma \mid X(\theta_1, \theta_2) \in \text{sf}(\varphi)\} \cup \{\langle\top, \top\rangle\}$, with $X \in \{Kc_i, N\}$ and $\text{sf}(\varphi)$ the set of subformulas of φ defined in the usual way. A canonical selection function sel_w^φ is a function that takes \mathfrak{M}_c^Γ, w and φ as input, returns a set $S' \subseteq S^\Gamma$, and is such that:*

- (1) $\text{sel}_w^\varphi(p) = \{w\}$;
- (2) $\text{sel}_w^\varphi(\neg\varphi_1) = \text{sel}_w^\varphi(\varphi_1)$;
- (3) $\text{sel}_w^\varphi(\varphi_1 \vee \varphi_2) = \text{sel}_w^\varphi(\varphi_1) \cup \text{sel}_w^\varphi(\varphi_2)$;
- (4) If $\llbracket X(\varphi_1, \varphi_2) \rrbracket^{\mathfrak{M}_c^\Gamma} \neq \emptyset$ and $\llbracket \varphi_1 \rrbracket^{\mathfrak{M}_c^\Gamma} = \emptyset$ for $X \in \{Kc_i, N\}$:
 $\text{sel}_w^\varphi(X(\varphi_1, \varphi_2)) = \{w\}$;
- (5) If $\llbracket X(\varphi_1, \varphi_2) \rrbracket^{\mathfrak{M}_c^\Gamma} \neq \emptyset$ and $\llbracket \varphi_1 \rrbracket^{\mathfrak{M}_c^\Gamma} \neq \emptyset$ for $X \in \{Kc_i, N\}$:
 $\text{sel}_w^\varphi(X(\varphi_1, \varphi_2)) = \{w_1, w_2\} \cup \text{sel}_{w_1}^\varphi(\varphi_1) \cup \text{sel}_{w_2}^\varphi(\varphi_2)$,
where w_1, w_2 are s.t. $(w_1, w_2) \in R_{\langle\varphi_1, \varphi_2\rangle}^\Gamma$;
- (6) If $\llbracket Kc_i(\varphi_1, \varphi_2) \rrbracket^{\mathfrak{M}_c^\Gamma} = \emptyset$ (note that $\llbracket \varphi_1 \rrbracket^{\mathfrak{M}_c^\Gamma} \neq \emptyset$):
For each $\langle\psi_1, \psi_2\rangle = \{a\} \in U^\Gamma(i) \cap \text{Act}_\varphi$:
(a) if $\llbracket \varphi_1 \rrbracket^{\mathfrak{M}_c^\Gamma} \not\subseteq \text{SE}(a)$:
add $\text{sel}_{w_1}^\varphi(\varphi_1) \cup \{w_1\}$ to $\text{sel}_w^\varphi(Kc_i(\varphi_1, \varphi_2))$,
where $w_1 \in \llbracket \varphi_1 \rrbracket^{\mathfrak{M}_c^\Gamma}$ and $w_1 \notin \text{SE}(a)$;
- (b) if $R_a^\Gamma(\llbracket \varphi_1 \rrbracket^{\mathfrak{M}_c^\Gamma}) \not\subseteq \llbracket \varphi_2 \rrbracket^{\mathfrak{M}_c^\Gamma}$:
add $\{w_1, w_2\} \cup \text{sel}_{w_1}^\varphi(\varphi_1) \cup \text{sel}_{w_2}^\varphi(\varphi_2)$ to $\text{sel}_w^\varphi(Kc_i(\varphi_1, \varphi_2))$,
where $w_1 \in \llbracket \varphi_1 \rrbracket^{\mathfrak{M}_c^\Gamma}$, $w_2 \in R_a^\Gamma(w_1)$ and $w_2 \notin \llbracket \varphi_2 \rrbracket^{\mathfrak{M}_c^\Gamma}$;
- (7) If $\llbracket N(\varphi_1, \varphi_2) \rrbracket^{\mathfrak{M}_c^\Gamma} = \emptyset$ (note that $\llbracket \varphi_1 \rrbracket^{\mathfrak{M}_c^\Gamma} \neq \emptyset$):
For each $\langle\psi_1, \psi_2\rangle = a \in N^\Gamma \cap \text{Act}_\varphi$:
(a) if $\llbracket \varphi_1 \rrbracket^{\mathfrak{M}_c^\Gamma} \not\subseteq \text{SE}(a)$:
add $\{w_1\} \cup \text{sel}_{w_1}^\varphi(\varphi_1)$ to $\text{sel}_w^\varphi(N(\varphi_1, \varphi_2))$,
where $w_1 \in \llbracket \varphi_1 \rrbracket^{\mathfrak{M}_c^\Gamma}$ and $w_1 \notin \text{SE}(a)$;
- (b) if $R_a^\Gamma(\llbracket \varphi_1 \rrbracket^{\mathfrak{M}_c^\Gamma}) \not\subseteq \llbracket \varphi_2 \rrbracket^{\mathfrak{M}_c^\Gamma}$:
add $\{w_1, w_2\} \cup \text{sel}_{w_1}^\varphi(\varphi_1) \cup \text{sel}_{w_2}^\varphi(\varphi_2)$ to $\text{sel}_w^\varphi(N(\varphi_1, \varphi_2))$,
where $w_1 \in \llbracket \varphi_1 \rrbracket^{\mathfrak{M}_c^\Gamma}$, $w_2 \in R_a^\Gamma(w_1)$ and $w_2 \notin \llbracket \varphi_2 \rrbracket^{\mathfrak{M}_c^\Gamma}$.

The overall idea is inspired by selection methods in modal logics [8], and in knowing how logic [2]. The selection function picks

enough states of the canonical model, in order to ensure the preservation of the truth of the subformulas of a given input formula. In addition, we also need to pick the proper set of normed plans and uncertainty sets. This is made precise in the following definition.

DEFINITION 5.2 (SELECTED MODEL). *Let \mathfrak{M}_c^Γ be the canonical model for an MCS Γ , w a state in \mathfrak{M}_c^Γ , and φ a formula. Let sel_w^φ be a selection function, we define the model selected by sel_w^φ as $\mathfrak{M}_w^\varphi = \langle S_w^\varphi, R_w^\varphi, V_w^\varphi, U_w^\varphi, N_w^\varphi \rangle$, where*

$$\begin{aligned} S_w^\varphi &= \text{sel}_w^\varphi(\varphi); \\ (R_w^\varphi)_{\langle\theta_1, \theta_2\rangle} &= R_{\langle\theta_1, \theta_2\rangle}^\Gamma \cap (S_w^\varphi \times S_w^\varphi), \text{ for each } \langle\theta_1, \theta_2\rangle \in \text{Act}_\varphi; \\ N_w^\varphi &= N^\Gamma \cap \text{Act}_\varphi; \\ (U_w^\varphi)(i) &= \{\{a\} \in U^\Gamma(i) \mid a \in \text{Act}_\varphi\} \cup \{\emptyset\}, \text{ for } i \in \text{Agt}; \\ V_w^\varphi &\text{ is the restriction of } V^\Gamma \text{ to } S_w^\varphi. \end{aligned}$$

PROPOSITION 5.1. *$\mathfrak{M}_w^\varphi = \langle S_w^\varphi, R_w^\varphi, V_w^\varphi, U_w^\varphi, N_w^\varphi \rangle$ is a U-NLTS. Moreover, $\Pi \in U_w^\varphi(i)$ implies $\Pi \subseteq N_w^\varphi$.*

PROOF. The structure $\langle S_w^\varphi, R_w^\varphi, V_w^\varphi \rangle$ is an LTS as $\text{sel}_w^\varphi(\varphi) \neq \emptyset$. Since we preserved $\langle\top, \top\rangle$ in N_w^φ , and (U_w^φ) is just a restriction of U^Γ to Act_φ , it is easy to see that it meets the conditions of Def. 3.7. Thus, \mathfrak{M}_w^φ is a U-NLTS. The last implication follows by definition. \square

Below we state the crucial property for characterizing the complexity of checking the satisfiability of DLKc-formulas.

PROPOSITION 5.2. *Let \mathfrak{M}_c^Γ be a canonical model, w a state in \mathfrak{M}_c^Γ and φ a formula. Let \mathfrak{M}_w^φ be the selected model by a selection function sel_w^φ . Then, $\mathfrak{M}_c^\Gamma, w \Vdash \varphi$ implies that for all ψ subformula of φ , and for all $v \in S_w^\varphi$, we have that $\mathfrak{M}_c^\Gamma, v \Vdash \psi$ iff $\mathfrak{M}_w^\varphi, v \Vdash \psi$. Moreover, \mathfrak{M}_w^φ is polynomial on the size of φ .*

PROOF (SKETCH). The proof of that \mathfrak{M}_w^φ preserves the satisfiability of formulas follows by a standard induction in the size of φ . It remains to show that \mathfrak{M}_w^φ is polynomial on the size of φ . The selection function adds states from \mathfrak{M}_c^Γ , only for each subformula of φ with Kc_i or N as the outermost connective. The number of states added at each time is polynomial in the size of φ . Hence, the size of S_w^φ is polynomial. Since $(U_w^\varphi)(i)$ and N_w^φ are also polynomial, the size of \mathfrak{M}_w^φ is polynomial in the size of φ . \square

In order to prove that the satisfiability problem is in NP, it remains to show that the model checking problem is in P (the proof is omitted due to lack of space, but it is similar to the one provided in [2] for an epistemic knowing how logic).

PROPOSITION 5.3. *The model checking problem for DLKc is in P.*

Now, we are in position to characterize the complexity of the satisfiability problem for DLKc.

THEOREM 2. *The satisfiability problem for DLKc is NP-complete.*

PROOF. Hardness follows from NP-completeness of propositional logic (a fragment of DLKc). By Prop. 5.2, each satisfiable formula φ has a model of polynomial size on φ . Thus, we can guess a polynomial model \mathfrak{M}, w , and test $\mathfrak{M}, w \Vdash \varphi$ (which can be done in polynomial time, due to Prop. 5.3). Therefore, the result follows. \square

6 REASONING ABOUT ABILITIES

So far, we studied a logical formalism that enables us to express and reason about normative courses of actions and responsibilities for agents. These notions are syntactically captured in our logic using the modalities N and Kc_i , respectively. From a semantic standpoint, these modalities are interpreted on a model relative to a set N of ‘normed’ plans. This raises the question of whether it is possible to reason in our logic about the possible plans the agents may engage on, i.e., expressing what an agent ‘can do’, independently of the given norms or even of their own individual perception. Having this new feature is useful for studying interactions between the given abilities and what agents can do according to the norms. To this end, in this section, we investigate the impact of adding a new modality (called S) to DLKc and refer to the resulting logic as DLKc⁺.

DEFINITION 6.1. *The language of DLKc⁺ is defined by:*

$$\rho, \psi ::= p \mid \neg\phi \mid \phi \vee \psi \mid S(\psi, \phi) \mid N(\psi, \phi) \mid Kc_i(\psi, \phi),$$

where: $p \in \text{Prop}$ and $i \in \text{Agt}$. Intuitively, a formula $S(\psi, \phi)$ is read as: “there is a course of action that brings about ϕ given ψ ”.

The semantic clause for the modality S is given below. It is worth noticing that this semantic clause is exactly the one for the knowing how modality of [32, 33]. As argued in [2], such a modality can be seen as an ability modality, rather than an epistemic one. We adhere to [2] and take S as an ability modality.

DEFINITION 6.2. *Let $\mathfrak{M} = \langle S, R, V, U, N \rangle$ be a U-NLTS, $s \in S$, and ϕ be a formula; Def. 3.8 is extended to account for the operator S as:*

$$\mathfrak{M}, s \models S(\psi, \phi) \quad \text{iff} \quad \text{exists } \pi \in \text{Act}^* \text{ such that}$$

- (i) $\llbracket \psi \rrbracket^{\mathfrak{M}} \subseteq \text{SE}(\pi)$ and
- (ii) $R_\pi(\llbracket \psi \rrbracket^{\mathfrak{M}}) \subseteq \llbracket \phi \rrbracket^{\mathfrak{M}}$.

The following example motivates the use of S .

EXAMPLE 6.1. *In the context of our FEEP example, the formula $S(\top, s)$ expresses that ‘there is a course of action that always (\top) allows agents to reach a safe location’, whereas $S(f \wedge c, s)$ states that ‘there is a course of action that always leads to a safe location in case of a fire event, provided there is the capacity (c) of doing so’. In the context of the LTS in Fig. 2, we have that*

$$(1) \mathfrak{M}, s_1 \models S(f \wedge c, s) \quad (2) \mathfrak{M}, s_1 \not\models S(\top, s).$$

For (1), we can take the plan $\pi_4 = \text{pull.alarm; use.ramp; call.999}$ as a witness. Notice that (2) holds, since in state s_2 , there is no plan for reaching a safe location (i.e., a state in which s holds). Intuitively, the LTS in Fig. 2 can be seen as illustrating the actions available in the case of a fire in a building. Item (1) tells us that in such a scenario it is possible to reach a safe location if there is the capacity to do so; whereas item (2) tells us that it is not always possible to reach a safe location, e.g., if we are trapped in an elevator.

An axiomatization for DLKc⁺ is obtained by adding the axiom schemas in Tab. 4 to the axiom system introduced in Tab. 3. Notice that, NS establishes the interaction between N and S . Intuitively, it tells us that whichever is regulated by norms is also feasible. This rules out normative systems in which certain norms are impossible –e.g., if we think of norms in terms of obligations, this axiom tells us that our logic adheres to the principle *impossibilia nulla obligatio est*, which states that impossible norms shall be excused. As one

Table 4: Additional axioms for DLKc⁺.

4SA $\vdash S(\psi, \phi) \rightarrow AS(\psi, \phi)$	NS $\vdash N(\psi, \phi) \rightarrow S(\psi, \phi)$
5SA $\vdash \neg S(\psi, \phi) \rightarrow A\neg S(\psi, \phi)$	EmpS $\vdash A(\psi \rightarrow \phi) \rightarrow S(\psi, \phi)$
	CompS $\vdash (S(\psi, \chi) \wedge S(\chi, \phi)) \rightarrow S(\psi, \phi)$

would expect, the converse of NS is not a theorem of the logic –i.e., it is possible for certain courses of actions not to be regulated by norms. Lastly, the axioms EmpS and CompS capture some intuitions behind the possible courses of actions. These axioms are present in the original proposal for a knowing how modality [32]. In particular, EmpS tells us that it is possible to turn universally valid implications into abilities by doing nothing (witnessed by the empty plan ϵ); whereas CompS tells us that courses of actions that have a common goal/context can be composed. In [2], it is argued that they account for some level of omniscience that one might disagree with. This challenge, however, is set aside here since S is taken not as an epistemic modality but as an *ability* modality.

THEOREM 3. *The axioms and rules in Tabs. 3 and 4 yield a sound and strongly complete axiom system for DLKc⁺ over U-NLTSs.*

We conclude with a comment on the computational behaviour of DLKc⁺. The semantics of S is exactly the one of Kh in [32, 33]. It is shown in [24], that the satisfiability problem is decidable for this logic (in fact, the result is proved for a more general logic). We claim that following ideas from [26], we can provide a complexity characterization for this logic, whose status is still an open problem.

7 FINAL REMARKS

We presented a deontic logic for modelling the notion of knowingly complying with a given set of norms. The logic features two modalities N and Kc_i , one refining the other. On the one hand, N models those abilities that are normed (i.e., those that are within the limits of the law). On the other hand, Kc_i is the modality of ‘knowingly complying’ (i.e., it models the consciousness of an agent when achieving a certain goal by using a normed course of action). We used a fire emergency evaluation plan (FEEP) as a running example to illustrate the components of the logic. Then, we introduced a sound and strongly complete axiom system to provide an account on how the modalities interact in the logic. Moreover, we showed that the satisfiability problem for the logic is NP-complete, relying on a small model property. Finally, we extended the logic with a modality S to capture the general abilities of the agents. We studied the effects of including such a modality. This result illustrates the flexibility of our framework, and shows how the three aspects in our deontic systems interact among each other.

There are several interesting directions for future work. First, it would be interesting to characterize the exact complexity of the satisfiability problem of the extended logic DLKc⁺. This would give us, as a by product, the exact complexity of the basic knowing how logic from [32, 34]. Second, by playing with the relation between the set of plans $U(i)$ of each agent and the set of norms N , it is possible to establish different levels of responsibility for the agents. This relates with the different notions of *knowingly doing* of [10–12]. Finally, our approach enables us to impose new restrictions on the different components of the model (or weakening them), and obtain new logics. It would be interesting to study these different systems in a deontic context.

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