Moving Arrows and Four Model Checking Results

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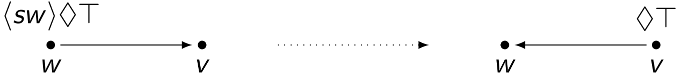
Modal logics: *"we like to talk about models"*

- Modal logics are known to describe models.
- Choose the right paintbrush:
  - $\Diamond\varphi$, $\Diamond^{-}\varphi$
  - $\mathsf{E}\varphi$
  - $\Diamond_{\geq n}\varphi$
  - $\Diamond^{*}\varphi$
  - ...
- Now, what about operators that can modify models?
  - Change the domain of the model.
  - Change the properties of the elements of the domain while we are evaluating a formula.
  - Evaluate φ after deleting/adding/swapping around an edge.

Logics that change the model 1/2

What about a swapping modal operator?



What happens when you add that to the basic modal logic?

Logics that change the model 2/2

What about:

- an edge-deleting modality?

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What about:

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- an edge-adding modality?

Sabotage Modal Logic [van Benthem 2002]

$$\mathcal{M}, w \models \langle gs \rangle \varphi \text{ iff } \exists \text{ pair } (u, v) \text{ of } \mathcal{M} \text{ such that } \mathcal{M}^-_{\{(u,v)\}}, w \models \varphi,$$

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What we know [Löding & Rohde 03]:

- Model checking is PSPACE-complete.
- Satisfiability is undecidable.

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- We will focus on operators that modify the accesibility relation.

Meet the new operators

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  - $\langle br \rangle\varphi$: add a new edge, traverse it, then evaluate φ.

Examples: no tree model property

Theorem

$\mathcal{ML}(\blacklozenge)$ lacks the tree model property, for $\blacklozenge \in \{\langle sw\rangle, \langle gs\rangle, \langle ls\rangle, \langle br\rangle\}$.

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Proof.

1. $\Box\bot \;\wedge\; \langle br\rangle\Box\bot$ — *w and $v \neq w$ are unconnected.*
2. $\Diamond\Diamond\top \;\wedge\; [gs]\Box\bot$ — *w is reflexive.*
3. $\Diamond\Diamond\top \;\wedge\; [ls]\Box\bot$ — *w is reflexive.*
4. $p \wedge (\bigwedge_{1\leq i\leq 3}\Box^i\neg p) \wedge \langle sw\rangle\Diamond\Diamond p$ — *w has a reflexive successor.* □

Bisimulations

We want to learn more about the models that these logics can describe.

So we need:

- Definition of $\blacklozenge$-bisimilarity.
- A bisimilarity theorem that says that two $\blacklozenge$-bisimilar models are undistinguishable by $\mathcal{ML}(\blacklozenge)$.

Conditions for ♦-bisimulations 1/2

always	(nontriv)	Z is not empty
always	(agree)	If $(w, S)Z(w', S')$, w and w' agree propositionally.
$\Diamond$	(zig)	If wSv, there is $v' \in W'$ s.t. $w'S'v'$ and $(v, S)Z(v', S')$
	(zag)	If $w'S'v'$, there is $v \in W$ s.t. wSv and $(v, S)Z(v', S')$
$\langle sw \rangle$	(sw-zig)	If wSv, there is $v' \in W'$ s.t. $w'S'v'$ and $(v, S^*_{vw})Z(v', S'^*_{v'w'})$
	(sw-zag)	If $w'S'v'$, there is $v \in W$ s.t. wSv and $(v, S^*_{vw})Z(v', S'^*_{v'w'})$

Conditions for ⧫-bisimulations 2/2

$\langle gs \rangle$	(gs-zig)	If vSu, there is $v', u' \in W'$ s.t. $v'S'u'$ and $(w, S^-_{vu})Z(w', S'^-_{v'u'})$
	(gs-zag)	If $v'S'u'$, there is $v, u \in W$ s.t. vSu and $(w, S^-_{vu})Z(w', S'^-_{v'u'})$
$\langle ls \rangle$	(ls-zig)	If wSv, there is $v' \in W'$ s.t. $w'S'v'$ and $(v, S^-_{wv})Z(v', S'^-_{w'v'})$
	(ls-zag)	If $w'S'v'$, there is $v \in W$ s.t. wSv and $(v, S^-_{wv})Z(v', S'^-_{w'v'})$
$\langle br \rangle$	(br-zig)	If $\neg wSv$, there is $v' \in W'$ s.t. $\neg w'S'v'$ and $(v, S^+_{wv})Z(v', S'^+_{w'v'})$
	(br-zag)	If $\neg w'S'v'$, there is $v \in W$ s.t. $\neg wSv$ and $(v, S^+_{wv})Z(v', S'^+_{w'v'})$

Invariance for Dynamic Logics

Theorem

For $\mathcal{ML}(\blacklozenge), \blacklozenge \in \{\langle sw \rangle, \langle gs \rangle, \langle ls \rangle, \langle br \rangle\}$, $\mathcal{M}, w \leftrightarroweq_{\mathcal{ML}(\blacklozenge)} \mathcal{M}', w'$ *implies* $\mathcal{M}, w \equiv_{\mathcal{ML}(\blacklozenge)} \mathcal{M}', w'$.

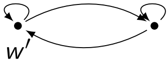
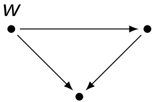
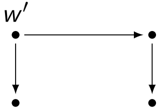
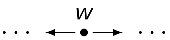
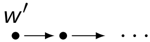
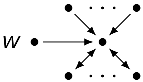
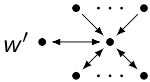
Comparing expressiveness

What if we want to show that all of these logics are uncomparable?

- Find two $\blacklozenge_1$-bisimilar models distinguishable by $\mathcal{ML}(\blacklozenge_2)$.
- Find two $\blacklozenge_2$-bisimilar models distinguishable by $\mathcal{ML}(\blacklozenge_1)$.

Then $\mathcal{ML}(\blacklozenge_1)$ and $\mathcal{ML}(\blacklozenge_2)$ are uncomparable.

Now let's have fun!

$\mathcal{M}$	$\mathcal{M}'$	Distinct by	Bisimilar for
		$\langle br\rangle\langle br\rangle\top$ $\langle gs\rangle\top$	$\mathcal{ML}(\langle ls\rangle)$ $\mathcal{ML}(\langle sw\rangle)$
		$\langle ls\rangle\Diamond\top$ $\langle gs\rangle\Diamond\top$	$\mathcal{ML}(\langle sw\rangle)$ $\mathcal{ML}(\langle br\rangle)$
		$\langle sw\rangle\langle sw\rangle\Diamond\Diamond\Diamond\Box\bot$ $[br][br]\bot$	$\mathcal{ML}(\langle gs\rangle)$ $\mathcal{ML}(\langle ls\rangle)$
		$\langle sw\rangle\Diamond\Box\bot$	$\mathcal{ML}(\langle br\rangle)$
		$\langle ls\rangle\Diamond\Box\bot$	$\mathcal{ML}(\langle gs\rangle)$

It all boils down to that...

Theorem

For all $\blacklozenge_1, \blacklozenge_2 \in \{\langle sw\rangle, \langle gs\rangle, \langle ls\rangle, \langle br\rangle\}$ *with* $\blacklozenge_1 \neq \blacklozenge_2$, $\mathcal{ML}(\blacklozenge_1)$ *and* $\mathcal{ML}(\blacklozenge_2)$ *are uncomparable.*

Other results: Model checking modal logics

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- For **g**lobal **s**abotage is PSPACE-complete [Löding & Rohde 03].
- Let us prove PSPACE-completeness for **l**ocal **s**abotage, **br**idge and **sw**ap logic.

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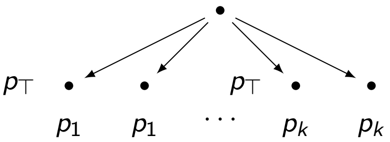

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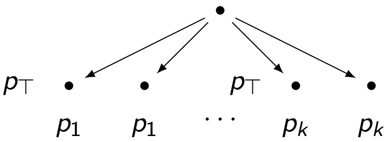

2. Build a $\mathcal{ML}(\langle sw \rangle)$ formula from a QBF as follows:

$$\begin{array}{ll} (\exists x_i.\alpha)' & = \langle sw \rangle (p_i \wedge \Diamond(\alpha)') \\ (x_i)' & = \neg\Diamond(p_i \wedge p_\top) \\ (\neg\alpha)' & = \neg(\alpha)' \\ (\alpha \wedge \beta) & = (\alpha)' \wedge (\beta)' \end{array}$$

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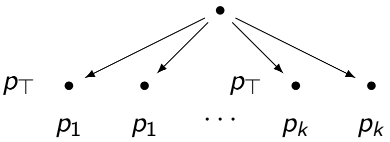

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3. α is true iff $\mathcal{M}_k, w \models (\alpha)'$

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Theorem

For $\blacklozenge \in \{\langle sw \rangle, \langle gs \rangle, \langle ls \rangle, \langle br \rangle\}$, model checking for any of the logics $\mathcal{ML}(\blacklozenge)$ is PSPACE-complete.

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- Further step: axiomatizations.