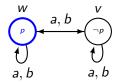
#### Logics with Copy and Remove

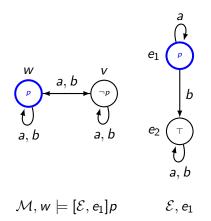
#### Carlos Areces <sup>1</sup>, Hans van Ditmarsch <sup>2</sup>, <u>Raul Fervari</u> <sup>1</sup> and François Schwarzentruber <sup>3</sup>

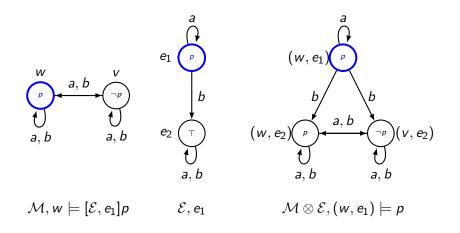
<sup>1</sup>FaMAF, Universidad Nacional de Córdoba & CONICET, Argentina <sup>2</sup>LORIA, CNRS - Université de Lorraine, France & IMSc, Chennai, India <sup>3</sup>ENS Rennes, France

WoLLIC 2014, Valparaíso, Chile



$$\mathcal{M}, w \models [\mathcal{E}, e_1]p$$





# 1/2

 An important application of modal logics is the representation of knowledge, belief and information change: Dynamic Epistemic Logics (DEL). [van Ditmarsch et al. 2007]

# 1/2

- An important application of modal logics is the representation of knowledge, belief and information change: Dynamic Epistemic Logics (DEL). [van Ditmarsch et al. 2007]
  - Public Announcement Logic.
  - Action Model Logic.

# 1/2

- An important application of modal logics is the representation of knowledge, belief and information change: Dynamic Epistemic Logics (DEL). [van Ditmarsch et al. 2007]
  - Public Announcement Logic.
  - Action Model Logic.
- Action Models: complicated update products which involve semantic objects into modalities.

# 1/2

- An important application of modal logics is the representation of knowledge, belief and information change: Dynamic Epistemic Logics (DEL). [van Ditmarsch et al. 2007]
  - Public Announcement Logic.
  - Action Model Logic.
- Action Models: complicated update products which involve semantic objects into modalities.
- Describe the information update by using more simple operations, inspired in Relation-Changing Modal Logics.

# 1/2

- An important application of modal logics is the representation of knowledge, belief and information change: Dynamic Epistemic Logics (DEL). [van Ditmarsch et al. 2007]
  - Public Announcement Logic.
  - Action Model Logic.
- Action Models: complicated update products which involve semantic objects into modalities.
- Describe the information update by using more simple operations, inspired in Relation-Changing Modal Logics.
  - [van Benthem 2005] [Löding & Rohde 2003] [Areces et al. 2012,13,14] [Fervari 2014]

Sabotage, Bridge, Swap.

Sabotage.

# 1/2

- An important application of modal logics is the representation of knowledge, belief and information change: Dynamic Epistemic Logics (DEL). [van Ditmarsch et al. 2007]
  - Public Announcement Logic.
  - Action Model Logic.

Sabotage, Bridge, Swap.

- Action Models: complicated update products which involve semantic objects into modalities.
- Describe the information update by using more simple operations, inspired in Relation-Changing Modal Logics.
  - [van Benthem 2005] [Löding & Rohde 2003] [Areces et al. 2012,13,14] [Fervari 2014]
- New approach: define product updates in terms of two primitives: copy & remove.

Sabotage.



Products in DEL do not always increase the size in the model.

- Products in DEL do not always increase the size in the model.
- We can see it as a two-step operation.

- Products in DEL do not always increase the size in the model.
- We can see it as a two-step operation.
- First, it generates the cartesian product between the epistemic and the action model.

- Products in DEL do not always increase the size in the model.
- We can see it as a two-step operation.
- First, it generates the cartesian product between the epistemic and the action model.
- After, it removes the inconsistent states.

- Products in DEL do not always increase the size in the model.
- We can see it as a two-step operation.
- First, it generates the cartesian product between the epistemic and the action model.
- After, it removes the inconsistent states.
- > Then, we introduce two dynamic modalities to capture this operation:

- Products in DEL do not always increase the size in the model.
- We can see it as a two-step operation.
- First, it generates the cartesian product between the epistemic and the action model.
- After, it removes the inconsistent states.
- > Then, we introduce two dynamic modalities to capture this operation:
  - Copy, replicates the original model keeping the accessibility relation between different copies.

- Products in DEL do not always increase the size in the model.
- We can see it as a two-step operation.
- First, it generates the cartesian product between the epistemic and the action model.
- After, it removes the inconsistent states.
- > Then, we introduce two dynamic modalities to capture this operation:
  - Copy, replicates the original model keeping the accessibility relation between different copies.
  - Remove, deletes paths on the accessibility relation.

# The Logic - Syntax

Given PROP, an infinite and countable set of propositional symbols, and AGT, a finite set of agents, let us define the set FORM of  $\mathcal{ML}(cp, rm)$ -formulas, together with a set PATH of path expressions.

# The Logic - Syntax

Given PROP, an infinite and countable set of propositional symbols, and AGT, a finite set of agents, let us define the set FORM of  $\mathcal{ML}(cp, rm)$ -formulas, together with a set PATH of path expressions.

 $\mathsf{FORM} ::= \bot \mid p \mid \neg \varphi \mid \varphi \land \varphi' \mid \Diamond_{\mathsf{a}} \varphi \mid \mathsf{rm}(\pi) \varphi \mid \mathsf{cp}(\bar{p}, q) \varphi,$ 

where  $\bar{p} = \langle p_1, \dots, p_n \rangle \in \mathsf{PROP}^n$  not appearing in any occurrence of cp in  $\varphi, q \in \bar{p}, a \in \mathsf{AGT}, \varphi, \varphi' \in \mathsf{FORM}$ , and  $\pi \in \mathsf{PATH}$ .

# The Logic - Syntax

Given PROP, an infinite and countable set of propositional symbols, and AGT, a finite set of agents, let us define the set FORM of  $\mathcal{ML}(cp, rm)$ -formulas, together with a set PATH of path expressions.

$$\mathsf{FORM} ::= \bot \mid p \mid \neg \varphi \mid \varphi \land \varphi' \mid \Diamond_{\mathsf{a}} \varphi \mid \mathsf{rm}(\pi) \varphi \mid \mathsf{cp}(\bar{p}, q) \varphi,$$

where  $\bar{p} = \langle p_1, \ldots, p_n \rangle \in \mathsf{PROP}^n$  not appearing in any occurrence of cp in  $\varphi$ ,  $q \in \bar{p}$ ,  $a \in \mathsf{AGT}, \varphi, \varphi' \in \mathsf{FORM}$ , and  $\pi \in \mathsf{PATH}$ .

$$\mathsf{PATH} ::= a \mid \pi; \pi' \mid \varphi?,$$

where  $a \in AGT$ ,  $\pi, \pi' \in PATH$  and  $\varphi$  is a **Boolean** formula.

$$\mathcal{P}^{\mathcal{M}}(a) = \{wau \mid (w, u) \in R_a\}$$

$$\begin{array}{lll} \mathcal{P}^{\mathcal{M}}(a) &= \{ wau \mid (w, u) \in R_a \} \\ \mathcal{P}^{\mathcal{M}}(\pi; \pi') &= \{ SwS' \mid Sw \in \mathcal{P}^{\mathcal{M}}(\pi) \text{ and } wS' \in \mathcal{P}^{\mathcal{M}}(\pi') \} \end{array}$$

$$\begin{array}{lll} \mathcal{P}^{\mathcal{M}}(a) &= \{wau \mid (w, u) \in R_a\} \\ \mathcal{P}^{\mathcal{M}}(\pi; \pi') &= \{SwS' \mid Sw \in \mathcal{P}^{\mathcal{M}}(\pi) \text{ and } wS' \in \mathcal{P}^{\mathcal{M}}(\pi')\} \\ \mathcal{P}^{\mathcal{M}}(\varphi?) &= \{w \mid \mathcal{M}, w \models \varphi\}. \end{array}$$

Let  $\mathcal{M} = \langle W, R, V \rangle$  a model and  $\pi \in \mathsf{PATH}$ . We define the set of  $\pi$ -paths  $\mathcal{P}^{\mathcal{M}}(\pi)$  of  $\mathcal{M}$  inductively as

$$\begin{array}{lll} \mathcal{P}^{\mathcal{M}}(a) &= \{wau \mid (w, u) \in R_a\} \\ \mathcal{P}^{\mathcal{M}}(\pi; \pi') &= \{SwS' \mid Sw \in \mathcal{P}^{\mathcal{M}}(\pi) \text{ and } wS' \in \mathcal{P}^{\mathcal{M}}(\pi')\} \\ \mathcal{P}^{\mathcal{M}}(\varphi?) &= \{w \mid \mathcal{M}, w \models \varphi\}. \end{array}$$

Let  $a \in AGT$ , we define edges<sub>a</sub>(P) that is the set of *a*-edges of the path P. Formally, edges<sub>a</sub>(P) = {(a, w, u) | wau is a subsequence of P}.

Let  $\mathcal{M} = \langle W, R, V \rangle$  a model and  $\pi \in \mathsf{PATH}$ . We define the set of  $\pi$ -paths  $\mathcal{P}^{\mathcal{M}}(\pi)$  of  $\mathcal{M}$  inductively as

$$\begin{array}{lll} \mathcal{P}^{\mathcal{M}}(a) &= \{wau \mid (w, u) \in R_a\} \\ \mathcal{P}^{\mathcal{M}}(\pi; \pi') &= \{SwS' \mid Sw \in \mathcal{P}^{\mathcal{M}}(\pi) \text{ and } wS' \in \mathcal{P}^{\mathcal{M}}(\pi')\} \\ \mathcal{P}^{\mathcal{M}}(\varphi?) &= \{w \mid \mathcal{M}, w \models \varphi\}. \end{array}$$

Let  $a \in AGT$ , we define edges<sub>a</sub>(P) that is the set of *a*-edges of the path P. Formally, edges<sub>a</sub>(P) = {(a, w, u) | wau is a subsequence of P}.

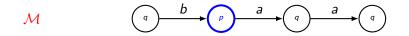
Let  $\mathcal{M} = \langle W, R, V \rangle$  a model and  $\pi \in \mathsf{PATH}$ . We define the set of  $\pi$ -paths  $\mathcal{P}^{\mathcal{M}}(\pi)$  of  $\mathcal{M}$  inductively as

$$\begin{array}{lll} \mathcal{P}^{\mathcal{M}}(a) & = & \{wau \mid (w, u) \in R_a\} \\ \mathcal{P}^{\mathcal{M}}(\pi; \pi') & = & \{SwS' \mid Sw \in \mathcal{P}^{\mathcal{M}}(\pi) \text{ and } wS' \in \mathcal{P}^{\mathcal{M}}(\pi')\} \\ \mathcal{P}^{\mathcal{M}}(\varphi?) & = & \{w \mid \mathcal{M}, w \models \varphi\}. \end{array}$$

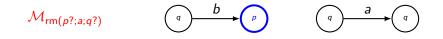
Let  $a \in AGT$ , we define edges<sub>a</sub>(P) that is the set of *a*-edges of the path P. Formally, edges<sub>a</sub>(P) = {(a, w, u) | wau is a subsequence of P}.

$$\begin{array}{ll} \mathcal{M}_{\mathsf{rm}(\pi)} = \langle W, R_{\mathsf{rm}(\pi)}, V \rangle, \text{ where} \\ R_{\mathsf{rm}(\pi)} &= R \setminus \bigcup_{\mathsf{a} \in \mathsf{AGT}, P \in \mathcal{P}^{\mathcal{M}}(\pi)} \mathsf{edges}_{\mathsf{a}}(P) \end{array}$$

$$\mathcal{M}_{\mathsf{rm}(\pi)} = \langle W, R_{\mathsf{rm}(\pi)}, V \rangle, \text{ where} \\ R_{\mathsf{rm}(\pi)} = R \setminus \bigcup_{a \in \mathsf{AGT}, P \in \mathcal{P}^{\mathcal{M}}(\pi)} \mathsf{edges}_{a}(P)$$



$$\mathcal{M}_{\mathsf{rm}(\pi)} = \langle W, R_{\mathsf{rm}(\pi)}, V \rangle, \text{ where} \\ R_{\mathsf{rm}(\pi)} = R \setminus \bigcup_{a \in \mathsf{AGT}, P \in \mathcal{P}^{\mathcal{M}}(\pi)} \mathsf{edges}_{a}(P)$$



 $\mathcal{M}_{cp(\bar{p})} = \langle W_{cp(\bar{p})}, R_{cp(\bar{p})}, V_{cp(\bar{p})} \rangle$ , where

$$\begin{array}{l} \mathcal{M}_{\mathsf{cp}(\bar{p})} = \langle W_{\mathsf{cp}(\bar{p})}, R_{\mathsf{cp}(\bar{p})}, V_{\mathsf{cp}(\bar{p})} \rangle, \text{ where} \\ W_{\mathsf{cp}(\bar{p})} &= \{(w,q) \mid w \in W \text{ and } q \in \bar{p} \} \end{array}$$

$$\begin{array}{ll} \mathcal{M}_{\mathsf{cp}(\bar{p})} = \langle W_{\mathsf{cp}(\bar{p})}, R_{\mathsf{cp}(\bar{p})}, V_{\mathsf{cp}(\bar{p})} \rangle, \, \text{where} \\ W_{\mathsf{cp}(\bar{p})} &= \{(w,q) \mid w \in W \, \, \text{and} \, \, q \in \bar{p} \} \\ R_{\mathsf{cp}(\bar{p})} &= \{(a,(w,q),(w',q')) \mid (a,w,w') \in R \} \end{array}$$

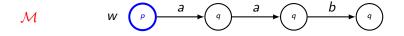
$$\begin{split} \mathcal{M}_{\mathsf{cp}(\bar{p})} &= \langle W_{\mathsf{cp}(\bar{p})}, R_{\mathsf{cp}(\bar{p})}, V_{\mathsf{cp}(\bar{p})} \rangle, \, \text{where} \\ W_{\mathsf{cp}(\bar{p})} &= \{(w,q) \mid w \in W \text{ and } q \in \bar{p}\} \\ R_{\mathsf{cp}(\bar{p})} &= \{(a,(w,q),(w',q')) \mid (a,w,w') \in R\} \\ V_{\mathsf{cp}(\bar{p})}(p) &= \{(w,q) \mid w \in V(p)\} \text{ for } p \neq q \end{split}$$

# The Logic - Paths & Updated Models 3/3

$$\begin{split} \mathcal{M}_{\mathsf{cp}(\bar{p})} &= \langle W_{\mathsf{cp}(\bar{p})}, R_{\mathsf{cp}(\bar{p})}, V_{\mathsf{cp}(\bar{p})} \rangle, \, \text{where} \\ W_{\mathsf{cp}(\bar{p})} &= \{(w,q) \mid w \in W \text{ and } q \in \bar{p}\} \\ R_{\mathsf{cp}(\bar{p})} &= \{(a,(w,q),(w',q')) \mid (a,w,w') \in R\} \\ V_{\mathsf{cp}(\bar{p})}(p) &= \{(w,q) \mid w \in V(p)\} \text{ for } p \neq q \\ V_{\mathsf{cp}(\bar{p})}(q) &= \{(w,q) \mid w \in W\}. \end{split}$$

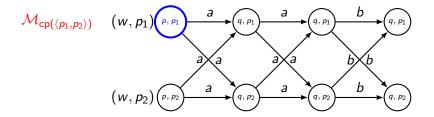
# The Logic - Paths & Updated Models 3/3

$$\begin{split} \mathcal{M}_{\mathsf{cp}(\bar{p})} &= \langle W_{\mathsf{cp}(\bar{p})}, R_{\mathsf{cp}(\bar{p})}, V_{\mathsf{cp}(\bar{p})} \rangle, \, \text{where} \\ W_{\mathsf{cp}(\bar{p})} &= \{(w,q) \mid w \in W \text{ and } q \in \bar{p}\} \\ R_{\mathsf{cp}(\bar{p})} &= \{(a,(w,q),(w',q')) \mid (a,w,w') \in R\} \\ V_{\mathsf{cp}(\bar{p})}(p) &= \{(w,q) \mid w \in V(p)\} \text{ for } p \neq q \\ V_{\mathsf{cp}(\bar{p})}(q) &= \{(w,q) \mid w \in W\}. \end{split}$$



# The Logic - Paths & Updated Models 3/3

$$\begin{split} \mathcal{M}_{\text{cp}(\bar{p})} &= \langle W_{\text{cp}(\bar{p})}, R_{\text{cp}(\bar{p})}, V_{\text{cp}(\bar{p})} \rangle, \text{ where} \\ W_{\text{cp}(\bar{p})} &= \{(w,q) \mid w \in W \text{ and } q \in \bar{p}\} \\ R_{\text{cp}(\bar{p})} &= \{(a,(w,q),(w',q')) \mid (a,w,w') \in R\} \\ V_{\text{cp}(\bar{p})}(p) &= \{(w,q) \mid w \in V(p)\} \text{ for } p \neq q \\ V_{\text{cp}(\bar{p})}(q) &= \{(w,q) \mid w \in W\}. \end{split}$$



#### The Logic - Semantics

Given a pointed model  $\mathcal{M}$ , w and a formula  $\varphi$  we say that  $\mathcal{M}$ , w satisfies  $\varphi$ , and write  $\mathcal{M}$ ,  $w \models \varphi$ , when

$$\begin{array}{lll} \mathcal{M},w\models p & \text{iff} & w\in V(p) \\ \mathcal{M},w\models \neg\varphi & \text{iff} & \mathcal{M},w\not\models\varphi \\ \mathcal{M},w\models \varphi\wedge\psi & \text{iff} & \mathcal{M},w\models\varphi \text{ and } \mathcal{M},w\models\psi \\ \mathcal{M},w\models \Diamond_a\varphi & \text{iff} & \text{for some } v\in W \text{ s.t. } (w,v)\in R_a, \ \mathcal{M},v\models\varphi \\ \mathcal{M},w\models \text{rm}(\pi)\varphi & \text{iff} & \mathcal{M}_{\text{rm}(\pi)},w\models\varphi \\ \mathcal{M},w\models \text{cp}(\bar{p},q)\varphi & \text{iff} & \mathcal{M}_{\text{cp}(\bar{p})},(w,q)\models\varphi. \end{array}$$

 $\varphi$  is satisfiable if for some pointed model  $\mathcal{M}, w$  we have  $\mathcal{M}, w \models \varphi$ .

It is enough to consider the conditions for the basic temporal logic  $\mathcal{ML}(\Diamond^{-1})$ :

It is enough to consider the conditions for the basic temporal logic  $\mathcal{ML}(\Diamond^{-1})$ :

If wZw' then:

(Atomic Harmony) for all  $p \in PROP$ ,  $w \in V(p)$  iff  $w' \in V'(p)$ ; (Zig) if  $(w, v) \in R_a$  then for some v',  $(w', v') \in R'_a$  and vZv';

(Zag) if  $(w', v') \in R'_a$  then for some  $v, (w, v) \in R_a$  and vZv'.

It is enough to consider the conditions for the basic temporal logic  $\mathcal{ML}(\Diamond^{-1})$ :

If wZw' then:

(Atomic Harmony) for all  $p \in PROP$ ,  $w \in V(p)$  iff  $w' \in V'(p)$ ;

(Zig) if  $(w, v) \in R_a$  then for some v',  $(w', v') \in R'_a$  and vZv'; (Zag) if  $(w', v') \in R'_a$  then for some v,  $(w, v) \in R_a$  and vZv'. (Zig<sup>-1</sup>) if  $(v, w) \in R_a$  then for some v',  $(v', w') \in R'_a$  and vZv'; (Zag<sup>-1</sup>) if  $(v', w') \in R'_a$  then for some v,  $(v, w) \in R_a$  and vZv'.

It is enough to consider the conditions for the basic temporal logic  $\mathcal{ML}(\Diamond^{-1})$ :

If wZw' then:

(Atomic Harmony) for all  $p \in PROP$ ,  $w \in V(p)$  iff  $w' \in V'(p)$ ;

(Zig) if  $(w, v) \in R_a$  then for some v',  $(w', v') \in R'_a$  and vZv'; (Zag) if  $(w', v') \in R'_a$  then for some v,  $(w, v) \in R_a$  and vZv'. (Zig<sup>-1</sup>) if  $(v, w) \in R_a$  then for some v',  $(v', w') \in R'_a$  and vZv'; (Zag<sup>-1</sup>) if  $(v', w') \in R'_a$  then for some v,  $(v, w) \in R_a$  and vZv'.

Theorem (Invariance under bisimulation.)

 $\mathcal{M}, w \cong_{\mathcal{ML}(\mathsf{cp},\mathsf{rm})} \mathcal{M}', w' \text{ implies } \mathcal{M}, w \equiv_{\mathcal{ML}(\mathsf{cp},\mathsf{rm})} \mathcal{M}', w'.$ 

# Action Models & Remove+Copy

We define a logic with two modalities to represent update products without using action models.

# Action Models & Remove+Copy

- We define a logic with two modalities to represent update products without using action models.
- We prove that there is a translation Tr from Action Model Logic to the logic with copy+remove, which preserves equivalence.

# Action Models & Remove+Copy

- We define a logic with two modalities to represent update products without using action models.
- We prove that there is a translation Tr from Action Model Logic to the logic with copy+remove, which preserves equivalence.

#### Theorem

Let  $\varphi$  an  $\mathcal{AML}$ -formula, then  $\varphi$  and  $\mathsf{Tr}(\varphi)$  are equivalent.

Complexity for some fragments:

Complexity for some fragments:

•  $\mathcal{ML}(cp)$  is PSPACE-complete.

Complexity for some fragments:

- $\mathcal{ML}(cp)$  is PSPACE-complete.
- $\mathcal{ML}(rm)$  is decidable.

Complexity for some fragments:

- $\mathcal{ML}(cp)$  is PSPACE-complete.
- $\mathcal{ML}(rm)$  is decidable.
- $\mathcal{ML}(cp, rm^{-})$  is NEXPTIME-complete.

► We define a logic with two dynamic operators: copy & remove.

- ► We define a logic with two dynamic operators: copy & remove.
- ▶ We embed DEL into this logic.

- ► We define a logic with two dynamic operators: copy & remove.
- ▶ We embed DEL into this logic.
- ► Also, we decompose copy and remove into simple action models.

- ► We define a logic with two dynamic operators: copy & remove.
- ▶ We embed DEL into this logic.
- Also, we decompose copy and remove into simple action models.
- ▶ We investigate computational complexity for fragments.

- ► We define a logic with two dynamic operators: copy & remove.
- We embed DEL into this logic.
- Also, we decompose copy and remove into simple action models.
- We investigate computational complexity for fragments.
- Limitation: we only consider Boolean preconditions. We will extend results for the full dynamic epistemic case.

- ► We define a logic with two dynamic operators: copy & remove.
- We embed DEL into this logic.
- Also, we decompose copy and remove into simple action models.
- We investigate computational complexity for fragments.
- Limitation: we only consider Boolean preconditions. We will extend results for the full dynamic epistemic case.

# Thanks!