Undecidability of Relation-Changing Modal Logics

Carlos Areces[1,2], Raul Fervari[1,2], Guillaume Hoffmann[1,2] & Mauricio Martel[3]

[1] FaMAF, Universidad Nacional de Córdoba, Argentina
[2] CONICET, Argentina
[3] Universität Bremen, Germany

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Modal logics: *"we like to talk about models"*

- Modal logics are known to describe models.
- Choose the right paintbrush:
  - $\Diamond\varphi$, $\Diamond^{-}\varphi$
  - $\mathsf{E}\varphi$
  - $\Diamond_{\geq n}\varphi$
  - $\Diamond^{*}\varphi$
  - ...
- Now, what about operators that can modify models?
  - Change the domain of the model.
  - Change the properties of the elements of the domain while we are evaluating a formula.
  - Evaluate φ after deleting/adding/swapping around an edge.

Logics that change the model 1/2

What about a swapping modal operator?

What happens when you add that to the basic modal logic?

Logics that change the model 2/2

What about:

- an edge-deleting modality?

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What about:

- an edge-deleting modality?
- an edge-adding modality?

Sabotage Modal Logic [van Benthem 2002]

$$\mathrm{M}, w \models \langle \mathrm{gsb} \rangle \varphi \quad \text{iff} \quad \exists \text{ pair } (u, v) \text{ of M such that } \mathrm{M}^{-}_{\{(u,v)\}}, w \models \varphi,$$

where $\mathrm{M}^{-}_{\{(u,v)\}}$ is M without the edge (u, v).

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What we know [Löding & Rohde 03]:

- Model checking is PSPACE-complete.
- Satisfiability is undecidable (multi-modal case, reduction from PCP).

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- We will focus on operators that modify the accesibility relation.

Meet the operators

Remember the Basic Modal Logic (ML).

- Syntax: propositional language + a modal operator $\Diamond$.

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- Translations into Hybrid Logic [AFHM14].

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- Two steps:
  - Adapt the undecidability proof of $\mathcal{ALC}$self to mono-modal memory logic.
  - Give a satisfiability preserving translation from memory logic into relation-changing logics.

Memory Logics in a nutshell

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ⓡ can be seen as marking a state as visited, and ⓚ as checking if a state has been visited.

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Proposition

The satisfiability problem of the memory logic $\mathcal{ML}(ⓡ, ⓚ)$ is undecidable.

Undecidability proofs

Each translation from ML(ⓡ, ⓚ) into relation-changing logics proceeds in two steps:

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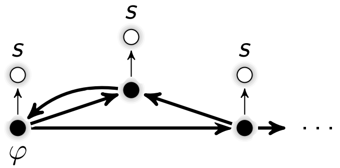
Proof

Satisfiability problem of $\mathrm{ML}(ⓡ, ⓚ) \Rightarrow$ satisfiability problem of $\mathrm{ML}(\blacklozenge)$, with $\blacklozenge \in \{\langle \mathrm{sb} \rangle, \langle \mathrm{gsb} \rangle, \langle \mathrm{br} \rangle, \langle \mathrm{gbr} \rangle, \langle \mathrm{sw} \rangle, \langle \mathrm{gsw} \rangle\}$.

Encoding ML(ⓡ, ⓚ) with global sabotage



$$Struct_{\langle \mathrm{gsb} \rangle}(\varphi) = \quad \neg s \wedge \bigwedge_{0 \leq i \leq \mathrm{md}(\varphi)} \Box^i(\neg s \ \rightarrow \Diamond s)$$

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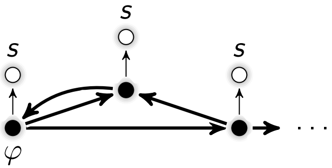

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Let φ be a ML(ⓡ, ⓚ)-formula, we define the translation into ML($\langle$gsb$\rangle$):

$$\begin{array}{lcl} \mathrm{Tr}_{\langle \mathrm{gsb} \rangle}(ⓚ) & = & \neg \Diamond s \\ \mathrm{Tr}_{\langle \mathrm{gsb} \rangle}(\Diamond \varphi) & = & \Diamond(\neg s \wedge \mathrm{Tr}_{\langle \mathrm{gsb} \rangle}(\varphi)) \\ \mathrm{Tr}_{\langle \mathrm{gsb} \rangle}(ⓡ \varphi) & = & \langle \mathrm{gsb} \rangle(\neg \Diamond s \wedge \mathrm{Tr}_{\langle \mathrm{gsb} \rangle}(\varphi)) \end{array}$$

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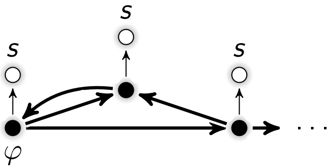

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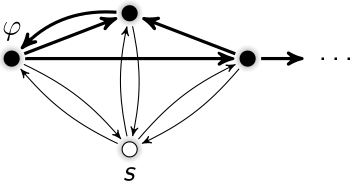
Then,

φ is satisfiable $\Leftrightarrow$ ($Struct_{\langle gsb \rangle}(\varphi) \wedge \mathrm{Tr}_{\langle gsb \rangle}(\varphi)$) is satisfiable

Encoding ML(ⓡ, ⓚ) with local sabotage



$$
\begin{aligned}
Struct_{\langle \mathrm{sb} \rangle} = \quad & s \wedge \Box\neg s \ \wedge \ \Box\Diamond s \ \wedge \ [\mathrm{sb}][\mathrm{sb}](s \rightarrow \Box\Diamond s) \\
& \wedge \Box[\mathrm{sb}](s \rightarrow \Diamond\Box\neg s) \\
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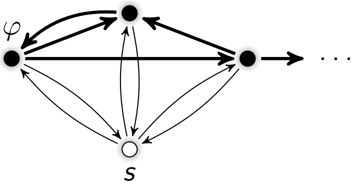

$$\begin{aligned}
Struct_{\langle sb\rangle} = \quad & s \wedge \Box\neg s \;\wedge\; \Box\Diamond s \;\wedge\; [sb][sb](s \to \Box\Diamond s) \\
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$\mathrm{Tr}_{\langle sb\rangle}(\varphi) = \Diamond(\varphi)'$, with:

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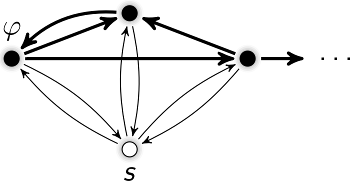

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Then,

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- For local swap we also need a spy point.
- Global cases and both versions of bridge are more similar to global sabotage.
- Proofs are adaptable for other versions of RCML (e.g., change adjacent edges but don't move).

Ending remarks

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- Finite satisfiability
  - undecidable for multi-modal sabotage logic
  - decidable for mono-modal local swap/sabotage.
- Proof systems.